PHY 6646 - Quantum Mechanics II - Spring 2022
Homework set # 8, due March 2

1. A particle of mass μ moves in a one-dimensional harmonic oscillator potential $V(x) = \frac{1}{2}\mu\omega^2 x^2$. Allowing for relativistic effects, the kinetic energy is $T = E - \mu c^2 = \sqrt{\mu^2 c^4 + p^2 c^2} - \mu c^2 \simeq \frac{p^2}{2\mu} - \frac{p^4}{8\mu^3 c^2}$. Treating the p^4 term as a perturbation, calculate the first order shift in the ground state energy.

2. Consider a particle of mass μ in the central force potential

$$\begin{aligned} V(r) &= -\frac{e^2}{r} && \text{for } 0 < r < R \\ &= -\frac{e^2}{r}\, \exp\left(-\lambda(r-R)\right) && \text{for } R < r < \infty \quad . \end{aligned} \tag{0.1}$$

This potential differs from the Coulomb potential only in the region $r > R$, where the Coulomb force is screened. The difference becomes negligible when $\lambda \to 0$. Consider this difference as a perturbation and calculate the first-order correction to the energy of the ground state of the hydrogen atom.

3. Problems 17.2.4, 17.2.5, 17.2.7, 17.2.8, 17.3.1 in Shankar's book.