PHY 6646 - Quantum Mechanics II - Spring 2022
Homework #10, due March 30

1. Consider a particle in the ground state of a box of length L. Argue on semiclassical grounds that the natural time period associated with this state is $T \simeq mL^2/\hbar\pi$. If the box expands asymmetrically to double its size, from $0 < x < L$ to $0 < x < 2L$, in time $\tau << T$ what is the probability of catching the particle in the ground state of the new box?

2. Problems 18.2.4, 18.2.5, 18.4.3, 18.4.4 in Shankar's book.

3. The four lowest energy states of the hydrogen atom are split in energy by the interaction between electron spin and proton spin. See problem 15.1.2. The groundstate has total spin $s = 0$. At an energy $\hbar\omega_0$, where $\omega_0 = (2\pi)$ 1.42 GHz, above the ground state are three states with $s = 1$. The electromagnetic radiation emitted or absorbed by transitions between the $s = 0$ and $s = 1$ states is commonly referred to as the "21 cm line"

a) A plane electromagnetic wave of wavevector $\vec{k}$, frequency $\omega = ck$, and energy per unit surface and unit time I, is incident upon a hydrogen atom in its ground state. The electromagnetic wave causes a time-dependent perturbation of the hydrogen atom

$$H^1(t) = -\gamma_e \vec{S}_e \cdot \vec{B}(\vec{0}, t) \tag{0.1}$$

where $\vec{S}_e$ is the spin of the electron, γ_e the electron gyromagnetic ratio, and $\vec{B}(\vec{0}, t)$ the magnetic field at the location of the atom. Eq. (0.1) neglects the much weaker interaction ($\gamma_p << \gamma_e$) of the electromagnetic field with the proton spin. Obtain the rate of transition of the atom from its $s = 0$ ground state to any of the three $s = 1$ states as a result of this perturbation. Does the rate depend on the direction of $\vec{k}$? Does it depend on the state of polarization of the electromagnetic wave?

b) The atom in its ground state is placed in a thermal bath of electromagnetic radiation at temperature T. Show that the rate at which it makes transitions to the $s = 1$ states is given by

$$R = \left(\frac{e}{m_e c}\right)^2 \frac{\hbar\omega_0^3}{c^3} \frac{1}{e^{\hbar\omega_0/k_b T} - 1} \tag{0.2}$$

where m_e is the electron mass, and k_B is Boltzmann's constant.

c) Verify that Eq. 0.2 is dimensionally correct.