Lecture 25

Friday, March 12, 2021

ELECTRONS IN MAGNETIC FIELDS

⇒ LANDAU LEVELS

QUASI-CLASSICAL EQ. OF MOTION

$$\frac{d\vec{r}}{dt} = \vec{V} = \frac{1}{\hbar} \nabla_k \mathcal{E}(\vec{k})$$

$$\hbar \frac{d\vec{k}}{dt} = -e\left(\frac{\vec{V}}{c} \times \vec{B}\right) \quad (\vec{B}\text{-FIELD})$$

⇒ ① ORBITS IN MAGNETIC LIE ON EQUAL ENERGY SURFACES.

② MOTION IN PLANE $\perp$ $\vec{B}$

USUALLY WANT $\omega_c \tau >> 1$ $\omega_c = \frac{eB}{m^* c}$ CYCLOTRON FREQ.

τ SCATTERING

$\Rightarrow$ (A) HIGH B

(B) LONG τ (SCATTERING IS WEAK $\Rightarrow$ "PURE" SAMPLES
$\Rightarrow$ LOW T)

EARTHS MAGNETIC
FIELD ~ 0.5 GAUSS

1 T = 10,000 GAUSS
TESLA

DIFFERENT TYPES OF MAGNETIC MEASUREMENTS.

(1) MAGNETO - RESISTANCE $\underline{\sigma_{xx}}$ ($\underline{\sigma_{xy}}$)

(2) ANOMOLOUS SKIN EFFECT (ABSORPTION OF MICROWAVES WHEN SKIN DEPTH IS SMALL ($\ell = v\tau$))

(3) CYCLOTRON RESONANCE

$$\omega_c = \frac{eB}{m^* c}$$

GIVES INFORMATION ON EFFECTIVE MASSES

(4) SHUBNIKOV-DE-HAAS EFFECT "OSCILLATIONS" IN RESISTIVITY WITH MAGNETIC FIELD

* (5) DE HAAS-VAN ALPHEN EFFECT "KITTEL" (BOOK)

OSCILLATIONS IN $\boxed{\chi \equiv \frac{dM}{dH}}$ PERIODIC IN $\frac{1}{B}$

MAG. SUSCEPTIBILITY

$$\vec{H} = \vec{B} - 4\pi\vec{M}$$

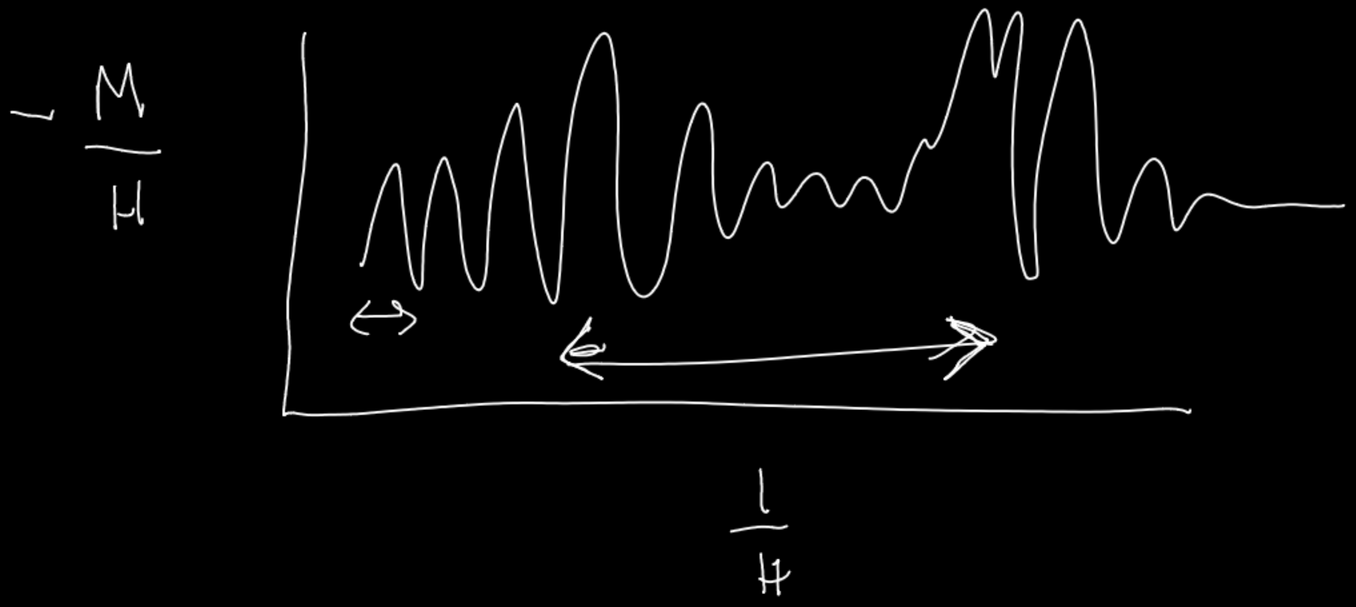
$\vec{M}$ = MAGNITIZATION = $\sum_i N_i \langle \vec{m}_i \rangle$

MAGNETIC MOMENT

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{J}$$

"FREE CURRENT"

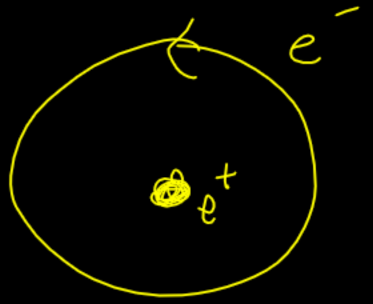
$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + 4\pi\,\nabla \times \vec{M}$$



WHERE DO OSCILLATIONS COME FROM?

ORBITS OF ELECTRONS IN A MAGNETIC FIELD

ARE $\boxed{\text{QUANTIZED.}}$ *



H ATOM

(THIS IS FOR "FREE" ELECTRONS)

$$\vec{P} = \vec{P}_{KIN} + \vec{P}_{FIELD} = \vec{P}_{KIN} + \frac{q\vec{A}}{c}$$

$\vec{A}$ = VECTOR POTENTIAL

$$\frac{\vec{P}_{KIN}^{\,2}}{2m} = \frac{(\vec{P} - \vec{P}_{FIELD})^2}{2m} = \frac{\left(\frac{\hbar\nabla}{i} - \frac{q\vec{A}}{c}\right)^2}{2m}$$

$\vec{B} = \nabla \times \vec{A}$

$$\oint \vec{p} \cdot d\vec{r} = (\underset{\text{INTEGER}}{n} + \lambda)\, 2\pi\hbar$$

$\boxed{\text{"BOHR - SOMMERFELD" QUANTIZATION}}$

"PHASE CORRECTION"

$= \frac{1}{2}$ FREE ELECTRONS

$$\oint \vec{p} \cdot d\vec{r} = \underbrace{\oint \hbar \vec{k} \cdot d\vec{r}}_{\text{PKW}} + \frac{q}{c} \underbrace{\oint \vec{A} \cdot d\vec{r}}_{\text{FIELD}}$$

$$\frac{d(\hbar k)}{dt} = \frac{q}{c}\left(\frac{d\vec{r}}{dt} \times \vec{B}\right) \Rightarrow \hbar \vec{k} = \frac{q}{c}(\vec{r} \times \vec{B}) + \text{CONSTANT}$$

$$\oint \hbar \vec{k} \cdot d\vec{r} = \frac{q}{c} \oint (\vec{r} \times \vec{B}) \cdot d\vec{r} = -\frac{q}{c} \vec{B} \cdot \underbrace{\oint \vec{r} \times d\vec{r}}_{2A}$$

$2A$ ← AREA

$$= -\frac{2q}{c} \Phi_B$$

MAGNETIC FLUX $= B \cdot A$ (AREA)

2ND TERM

$$\frac{q}{c} \oint \vec{A} \cdot d\vec{r} = \oint (\underbrace{\nabla \times \vec{A}}_{\vec{B}}) \cdot d\vec{s} = +\frac{q}{c} \Phi_B$$

↑ AREA ELEMENT

$$\boxed{-\frac{q}{c}\Phi_B = (n+\lambda)\,2\pi\hbar}$$

MAGNETIC FLUX IS QUANTIZED!!