

Lecture 15

Wednesday, Feb. 17, 2021

AT A BOUNDARY  $\vec{k} = \frac{\hbar}{2m} \vec{G}$   $\vec{G} \in R.L.$   
NEAR  $\vec{k} \approx \frac{\hbar}{2m} \vec{G}$

$$\psi(x) = c(k) e^{ikx} + c(k-G) e^{i(k-G)x}$$

$$\Rightarrow \begin{pmatrix} \lambda_{k-\varepsilon} & U \\ U & \lambda_{k-G} - \varepsilon \end{pmatrix} \begin{pmatrix} c(k) \\ c(k-G) \end{pmatrix} = 0$$

$$U = U_G$$

$$\lambda_k = \frac{\hbar^2 k^2}{2m}$$

$$\left| \begin{pmatrix} \lambda_{k-\varepsilon} & U \\ U & \lambda_{k-G} - \varepsilon \end{pmatrix} \right| = 0 \Rightarrow \varepsilon$$

$$\epsilon^2 - \underbrace{(\lambda_k + \lambda_{k-G})}_{\text{TRACE}} \epsilon + \underbrace{(\lambda_k \lambda_{k-G} - v^2)}_{\text{DET}} = 0$$

$$\epsilon = \frac{1}{2} (\lambda_{k-G} + \lambda_k) \pm \left[ \frac{1}{4} (\lambda_{k-G} - \lambda_k)^2 + v^2 \right]^{\frac{1}{2}}$$

$$\underbrace{\frac{1}{2} \frac{\hbar^2 (k-G)^2}{2m}}_{\text{}} \quad \underbrace{\frac{1}{2} \frac{\hbar^2 k^2}{2m}}_{\text{}}$$

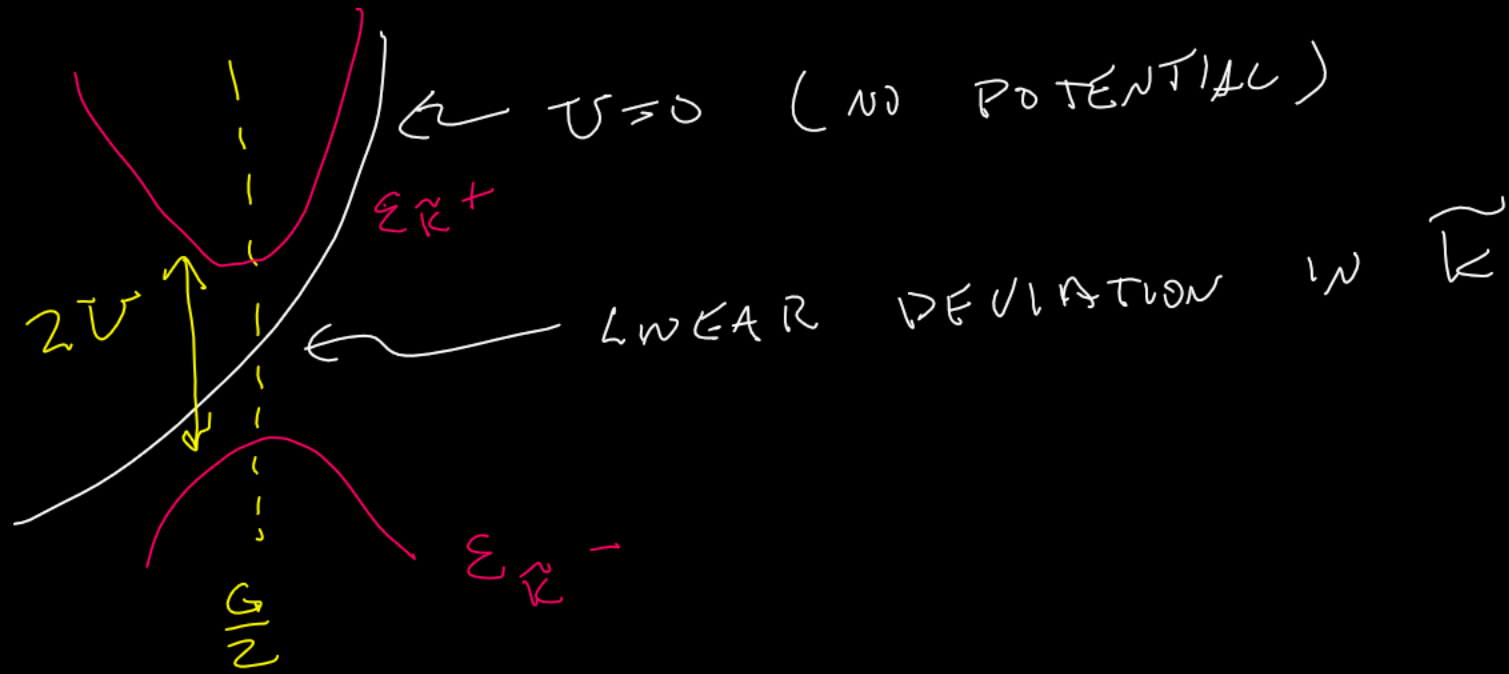
$$\tilde{k} = k - \frac{G}{2}$$

BINOMIAL EXPANSION

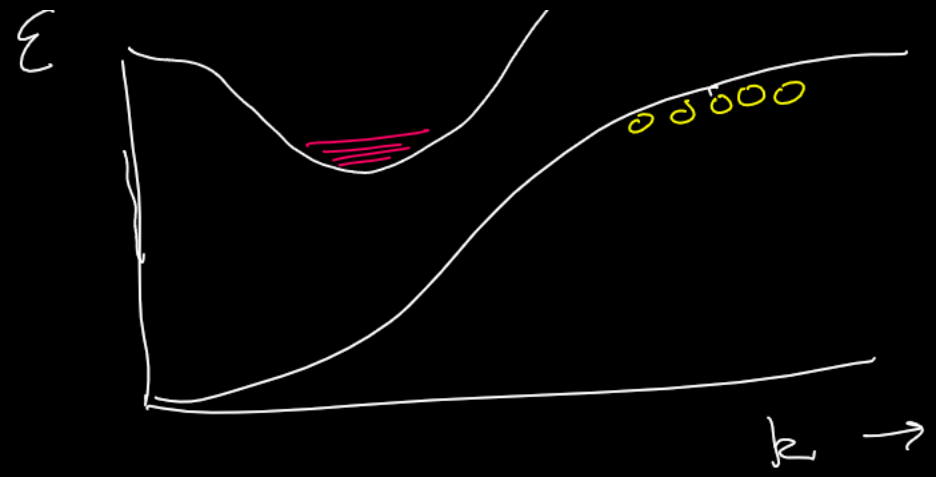
$$\epsilon_{\tilde{k}}(\pm) \approx \underbrace{\epsilon(\pm)}_{\text{BEFORE}} + \frac{\hbar^2 \tilde{k}^2}{2m} \left( 1 \pm \frac{v}{v} \right)$$

$$E_{\vec{k}}(\pm) \approx \underbrace{E(\pm)}_{\substack{\text{BEFORE} \\ (k = G/2)}} + \frac{\hbar^2 \vec{k}^2}{2m} \left( 1 \pm \frac{2\lambda}{v} \right)$$

$\uparrow$   
 $\sim \vec{k}^2 \rightarrow \underline{\text{QUADRATIC}}$

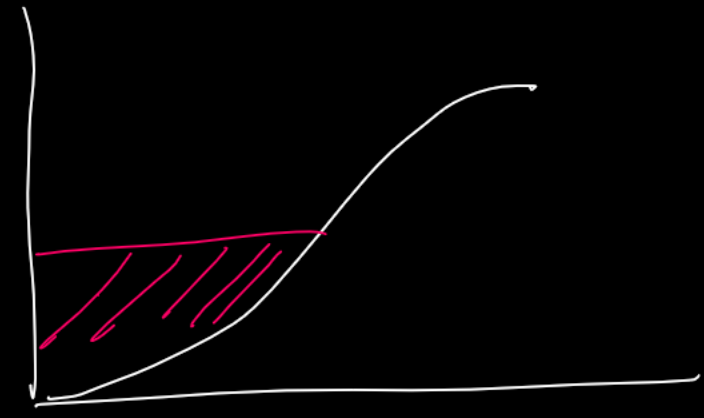


SEMI METAL



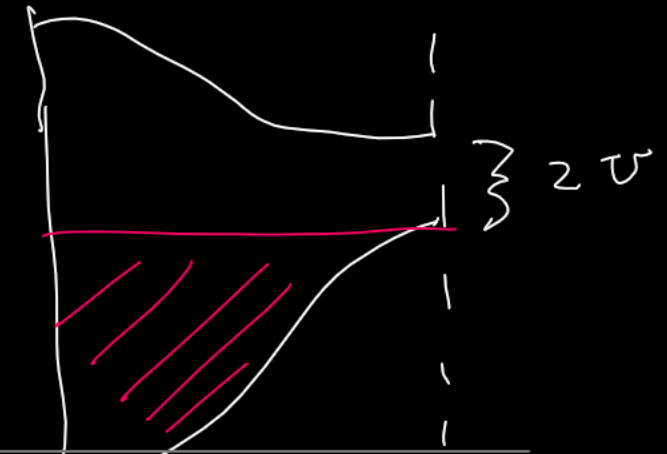
OVERLAPPING  
PARTIAL FILLED  
BANDS

METAL



PARTIALLY  
FILLED BAND

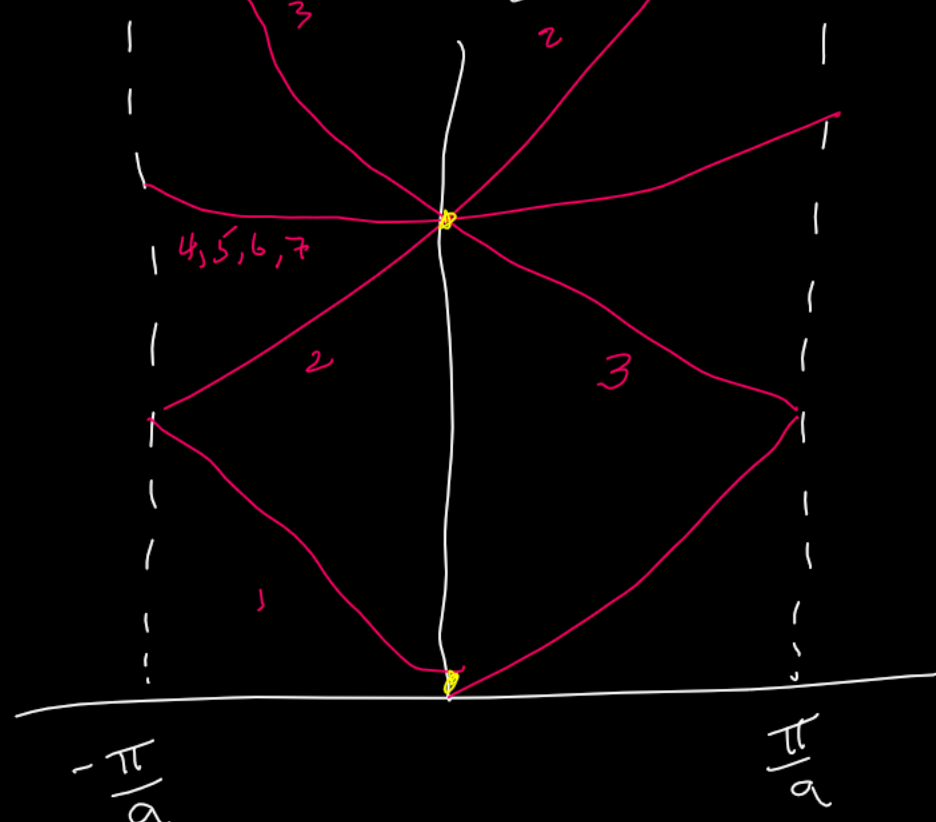
INSULATOR



COMPLETELY  
FILLED  
BAND

① X'TAL SYMMETRY  $V=0$

$$E(k_x, k_y, k_z) = \frac{\hbar^2}{2m} (\vec{k} + \vec{G})^2$$



$-\frac{\pi}{a}$   
 $G(a/2\pi)$   
 $0, 0, 0$

$\frac{2mE(0,0,0)}{\hbar^2}$   
 $0$

$\vec{G} \in R.L.$

SIMPLE CUBIC

$$E = \frac{\hbar^2}{2m} \left\{ (k_x + G_x)^2 + (k_y + G_y)^2 + (k_z + G_z)^2 \right\}$$

$\frac{100}{\hbar^2} (\hat{x})$

$E(k_x, 0, 0)$  (100) DIR.

$k_x^2$

BAND

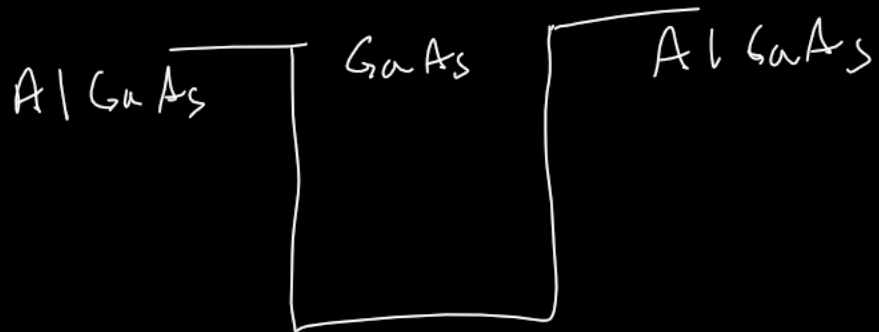


INHOMOGENEOUS

AlGaAs

GaAs

HETERO JUNCTION



"QUANTUM WELL"



SUPER LATTICE

