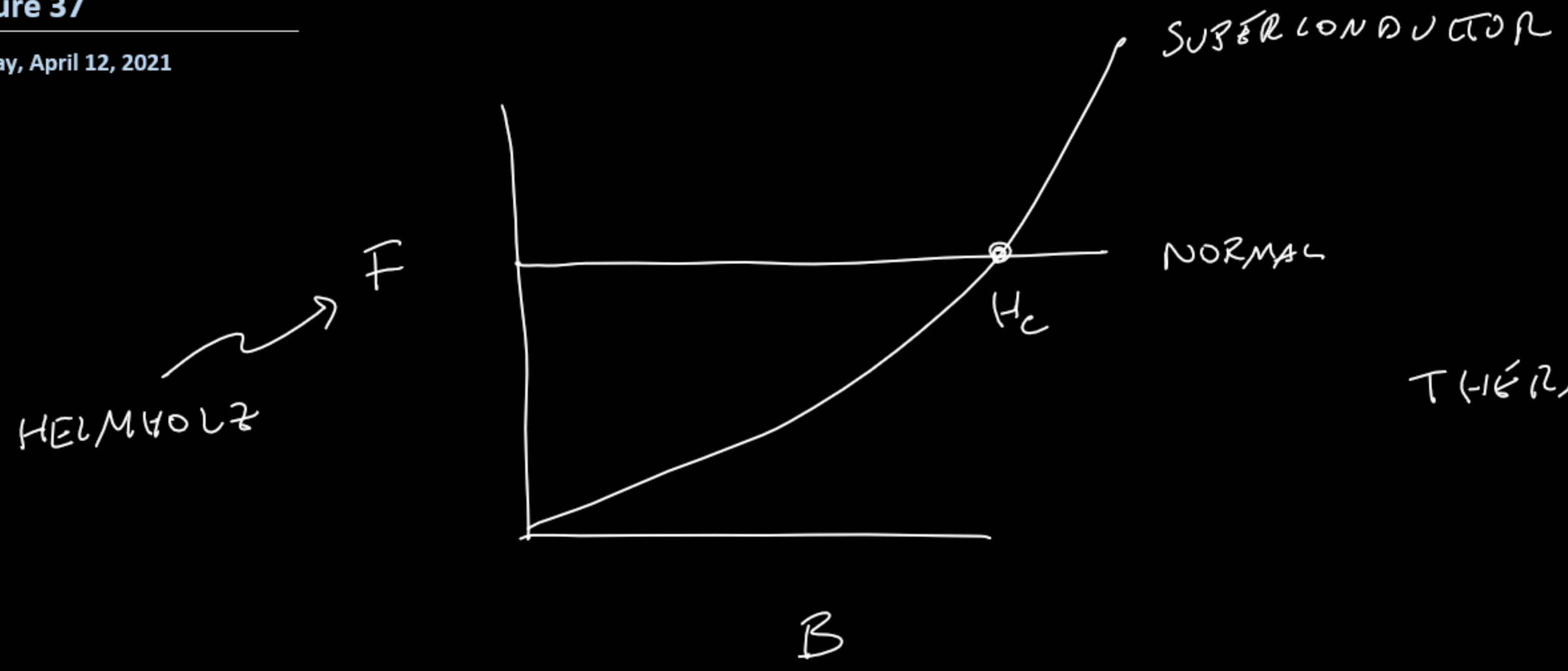
Lecture 37

Monday, April 12, 2021



THERMO $\Rightarrow$ H_c, I_c

$$\left.\frac{\partial F_N}{\partial T}\right|_{T_c} = \left.\frac{\partial F_S}{\partial T}\right|_{T_c} \Rightarrow$$

NO LATENT HEAT

S.C. TRANSITION IS 2ND ORDER

(1ST ORDER $\Rightarrow$ LATENT HEAT (ICE – LIQUID H_2O
LIQUID H_2O $\rightarrow$ STEAM)

LONDON THEORY (MEISSNER EFFECT)

(PHENOMONOLOGICAL THEORY)

$$\frac{d\vec{P}}{dt} = -\frac{\vec{P}}{\underset{\text{SCATTERING TIME}}{\tau}} + \underset{\text{EXT FORCE}}{\vec{F}} \qquad \text{(DRUDE)}$$

STEADY STATE: $\frac{d\vec{P}}{dt} = 0$

$$m\vec{V} = \vec{F}\tau = -e\vec{E}\tau$$

$$\underset{\text{NORMAL}}{\vec{J}_n} = -ne\vec{V} = +\underbrace{\frac{ne^2\tau}{m}} \vec{E} \qquad \text{NORMAL STATE}$$

τ NORMAL

$$\sigma = \frac{ne^2\tau}{m}$$

S.C. STATE GUESS:

$$m\frac{d\vec{V}_s}{dt} = -e\vec{E} \quad \text{(NO DAMPING)}$$

$\Longrightarrow$ $\vec{V}_s$ INCREASES IN TIME

$$\vec{J}_s = -n_s e\vec{V}_s \Longrightarrow \frac{d\vec{J}_s}{dt} = \frac{ne^2}{m}\vec{E}$$

SUPER C. DENSITY

(NOTE: F.T.

$$-i\omega\vec{J}_s = \frac{ne^2}{m}\vec{E}$$

$$\sigma(\omega) = \frac{+i n_s e^2}{m\omega}$$ S.C., A.C. CONDUCTIVITY

BEFORE: $\sigma(\omega) = \dfrac{\sigma(0)}{1 - i\omega\tau}$ (LIMIT $\tau \to \infty$)

$$\hookrightarrow = \frac{i n_s e^2}{m\omega}$$

$$\boxed{\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t}}$$ * *

$\vec{A}$ = VECTOR POTENTIAL

MAXWELL'S EQ.

$\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \nabla \times \vec{A}$

VECTOR POTENTIAL

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \Rightarrow \nabla \times \left(\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right) = 0$$

$$\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t} = -\nabla \phi$$

SCALAR POTENTIAL

$$\boxed{\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla \phi}$$

(MAXWELL'S EQ.)

$\vec{E}$ EITHER ∇ SCALAR POT.

OR $\frac{\partial}{\partial t}$ VECTOR

$$\frac{d\vec{J}_s}{dt} = -\frac{n_s e^2}{mc}\frac{\partial \vec{A}}{\partial t} \Rightarrow$$

$$\boxed{\vec{J}_s = -\frac{n_s e^2}{mc}\vec{A}}$$

SC. CURRENT DENSITY IS PROPORTIONAL TO VECTOR POTENTIAL.

THIS GIVE US THE MEISSNER EFFECT.

$$\nabla \times \vec{J}_s = -\frac{ne^2}{mc}\underbrace{(\nabla \times \vec{A})}_{\vec{B}} = -\frac{ne^2}{mc}\vec{B}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J}$$ (MAXWELL'S EQ. "AMPERES LAW)

$$\underbrace{\nabla \times (\nabla \times \vec{B})}_{-\nabla^2 \vec{B}} = -\frac{4\pi}{mc^2} n e^2 \vec{B}$$

EQ. FOR $\vec{B}$

"VECTOR" LAPLACIAN

$$\Rightarrow \quad \boxed{\nabla^2 \vec{B} = \frac{n e^2 4\pi}{mc^2} \vec{B}}$$

<u>UNIFORM</u> SYSTEM $\nabla^2 \vec{B} = 0 \Rightarrow \boxed{\vec{B} = 0}$

ONLY SOLUTION

NON-UNIFORM

HALF-SPACE

NORMAL STATE OR VACUUM

S.C.

$x=0$

$$B(x) = B(0)\, e^{-x/\lambda_L}$$

DECAYS EXPONENTIALLY

$$\boxed{\lambda_L = \left(\frac{mc^2}{4\pi n e^2}\right)^{\frac{1}{2}} = \frac{c}{\omega_p}}$$

"LONDON PENETRATION DEPTH"

MAGNETIC FIELD DECAYS EXPONENTIALLY AWAY FROM SURFACE

$$\boxed{\lambda_L \sim 100\,\text{Å}}$$

ANOTHER LENGTH SCALE IS "COHERENCE" LENGTH

ξ "XI"

$\xi_0 \equiv$ INTRINSIC COHERENCE LENGTH

$\xi = (\xi_0 \ell)^{\frac{1}{2}} =$ COHERENCE LENGTH

ℓ — MEAN FREE PATH IN NORMAL METAL

$\xi_0 \Rightarrow$ LENGTH SCALE FOR MODULATION OF THE SUPERCONDUCTING WAVE FUNCTION

$$\varphi(x) \sim e^{ikx} \Rightarrow |\varphi(x)|^2 \rightarrow \text{CONSTANT (NO MODULATION)}$$

$$\varphi(x) = \frac{1}{\sqrt{2}}\left(e^{i(k+q)x} + e^{ikx}\right)$$

$$\Rightarrow \quad |\varphi(x)|^2 = \frac{1}{2}\left(2 + e^{iqx} + e^{-iqx}\right)$$

$$= 1 + \cos qx \quad \Longrightarrow$$ MODULATION IN CARRIER DENSITY

$$\widehat{K.E.} = -\frac{\hbar^2 \nabla^2}{2m}$$

$$\langle K.E. \rangle = \int dx\, \varphi^* \left(-\frac{\hbar^2 \nabla^2}{2m}\right) \varphi =$$

$$\left(\frac{1}{2}\right) \frac{\hbar^2}{2m} \left((k+q)^2 + k^2 \right) \approx \frac{\hbar^2}{2m} \left(k^2 + \underline{\underline{kq}} + \underbrace{q^2}_{\text{DROP}} \right)$$

EXTRA K.E. DUE TO THE MODULATION

$$\approx \underbrace{\frac{\hbar^2}{2m} k q}_{} \qquad (q \ll k)$$

IF THIS ENERGY > "E_g" (CONDENSATE ENERGY)

$$\frac{\hbar^2}{2m} k_f q_0 = E_g \Rightarrow \boxed{q_0 = \frac{E_g}{\frac{\hbar^2 k_f}{2m}}}$$

q_0 ← CRITICAL q

$$\boxed{\xi_0 \equiv \frac{1}{q_0} = \frac{\hbar^2 k_f}{2mE_g}} \quad \text{COHERENCE LENGTH}$$

		T_c
Sn	2300 Å	3.7 K
Al	16000 Å	1.1 K
Nb	380 Å	9.5 K

"DIRTY" S.C.

$$\xi = (\xi_0 \, l)^{\frac{1}{2}}$$

$$\lambda \approx \lambda_L (\xi_0 / l)^{\frac{1}{2}}$$

$$\Rightarrow \underset{\text{KAPPA}}{K} \equiv \lambda / \xi = \frac{\lambda_L}{l}$$