HW 2 Solutions

Chap. 6, problem 6.

⑥ IF THE FIELD VARIES AS $E e^{-i\omega t}$, WE CAN ASSUME A SOLUTION OF THE FORM $V e^{-i\omega t}$ (NEGLECTING AN INITIAL TRANSIENT). SO THAT THE DRIFT VELOCITY EQUATION BECOMES:

$$-i\omega m V e^{-i\omega t} + \frac{mV}{\tau} e^{-i\omega t} = -eEe^{-i\omega t}$$

SOLVING FOR V:

$$V = \frac{-eE}{m\left(\frac{1}{\tau} - i\omega\right)} = \frac{-e\tau E}{1 - i\omega\tau}$$

SINCE $j = n(-e)V$

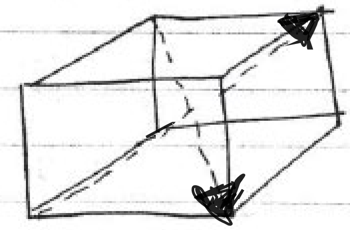
AND

$$\sigma(\omega) = \frac{E_\omega}{V_\omega}$$, IT FOLLOWS

$$\boxed{\sigma(\omega) = \underbrace{\frac{ne^2\tau}{m}}\left(\frac{1}{1-i\omega\tau}\right) = \sigma_0 \frac{1+i\omega\tau}{1+(\omega\tau)^2}}$$

Chapter 1:

(1) BODY DIAGONAL ONE HAS



$\vec{a}_1 = \hat{x} + \hat{y} + \hat{z}$

BODY DIAGONAL TWO IS GIVEN BY

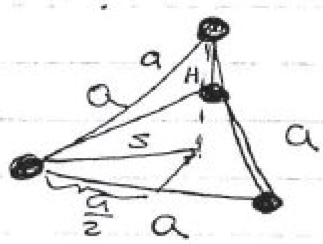
$\vec{a}_2 = -\left(-\hat{x} + \hat{y} + \hat{z}\right)$ (GOES DOWN)

SO $\cos\theta = \dfrac{\vec{a}_1 \cdot \vec{a}_2}{|\vec{a}_1||\vec{a}_2|} = -\dfrac{1}{3}$

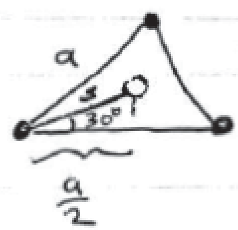
OR $\boxed{\theta = 109.47^\circ}$

(3) THE HCP WITH CLOSEST PACKING WILL OCCUR WHEN THE LAYERS ARE STACKED "TETRAHEDRALLY"

SO $C = 2H$ WHERE H IS THE HEIGHT OF THE TETRAHEDRON



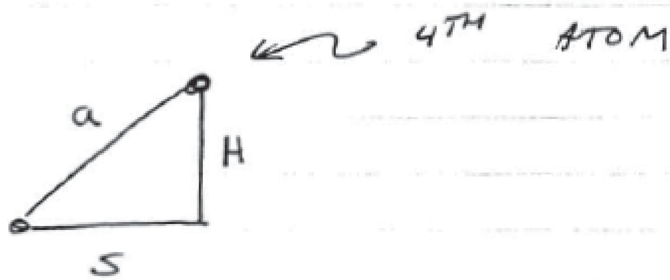
LOOKING AT THE PROJECTION OF THE FOURTH ATOM IN THE PLANE OF THE OTHER 3:



WE SEE THAT

$$S = \frac{\frac{a}{2}}{\cos 30^\circ} = \frac{a}{\sqrt{3}}$$

AND LOOKING AT THE VERTICAL PLANE



WE SEE $H = \sqrt{a^2 - S^2}$

OR $H = \sqrt{2/3}\ a$

AND $C = 2H \Rightarrow$

$$\boxed{C = \sqrt{\frac{8}{3}}\ a}$$

(4)

(A) $|F|^2 =$

$$\frac{\left(1-e^{-iM\vec{a}\cdot\Delta\vec{k}}\right)\left(1-e^{+iM\vec{a}\cdot\Delta\vec{k}}\right)}{\left(1-e^{-i\vec{a}\cdot\Delta\vec{k}}\right)\left(1-e^{+i\vec{a}\cdot\Delta\vec{k}}\right)}$$

$$= \frac{-\frac{1}{4}\left(e^{+iM\frac{\vec{a}\cdot\Delta\vec{k}}{2}}-e^{-iM\frac{\vec{a}\cdot\Delta\vec{k}}{2}}\right)^2}{-\frac{1}{4}\left(e^{+i\frac{\vec{a}\cdot\Delta\vec{k}}{2}}-e^{-i\vec{a}\cdot\Delta\vec{k}/2}\right)^2}$$

$$= \boxed{\frac{\sin^2 M\frac{\vec{a}\cdot\Delta\vec{k}}{2}}{\sin^2\frac{\vec{a}\cdot\Delta\vec{k}}{2}}}$$

(B) 1^{ST} MIN WHEN $\frac{1}{2}M(\vec{a}\cdot\Delta\vec{k}) =$

IF $\vec{a}\cdot\Delta k = 2\pi h + \epsilon$

$\Rightarrow \quad \frac{M\epsilon}{2} = \pi$ OR

$$\boxed{\varepsilon = 2\pi / M}$$

(5) TAKING THE CONVENTIONAL CUBE AS THE CELL, THE FCC

(A) LATTICE HAS ~~ ~~ ATOMS AT $(a = 1)$

$\vec{b}_1 = (0,0,0)$
$\vec{b}_2 = (\frac{1}{2}, 0, \frac{1}{2})$
$\vec{b}_3 = (\frac{1}{2}, \frac{1}{2}, 0)$
$\vec{b}_4 = (0, \frac{1}{2}, \frac{1}{2})$

IF WE TAKE THE DIAMOND STRUCTURE WE ADD AN ADDITIONAL POINT ~~ ~~ FOR EACH $\vec{b}_i$ AT $\vec{b}_i + (\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$

THE STRUCTURE FACTOR FOR DIAMOND IS:

$$S_{DIAM}(G) = \sum_{\vec{b}_i} \sum_{\vec{B} = (0,0,0),\ (\frac{1}{4},\frac{1}{4},\frac{1}{4})} e^{-i\left(\vec{G}\cdot(\vec{b}_i + \vec{B})\right)}$$

WE SEE THAT THE STRUCTURE FACTOR FACTORIZES INTO A PRODUCT, USING $\vec{G} = 2\pi(V_1, V_2, V_3)$ WITH V_1, V_2, V_3 INTEGERS, (RECALL WE ARE TAKING $a = 1$)

$$S(V_1 V_2 V_3) = S(FCC) \times S(BASIS)$$

WITH

$$S(FCC) = 1 + e^{-i\pi(v_1+v_2)} + e^{-i\pi(v_2+v_3)} + e^{-i\pi(v_3+v_1)}$$

$$= 1 + (-1)^{(v_1+v_2)} + (-1)^{(v_2+v_3)} + (-1)^{(v_3+v_1)}$$

AND

$$S(BASIS) = 1 + e^{-i\frac{\pi}{2}(v_1+v_2+v_3)}$$

(B) NOW $S(FCC) = 0$ UNLESS
ALL v_i ARE ALL EVEN OR ODD

ALSO;
$S(BASIS) \neq 0$ IF ALL ARE ODD.

IF ALL ARE EVEN,
$S(BASIS) = 0$ IF $v_1 + v_2 + v_3 = 2 \pmod 4$

BUT $S(BASIS) \neq 0$ IF $v_1 + v_2 + v_3 = 4n$
WHERE n IS AN INTEGER.

$\therefore$ REQUIREMENTS FOR $S \neq 0$ ARE:

1) v_1, v_2, v_3 ALL ODD

OR

2) v_1, v_2, v_3 ALL EVEN AND

$$v_1 + v_2 + v_3 = 4n$$