HW Set #1

PROBLEM SET #1

1) $U = \int_0^{\varepsilon_F} \varepsilon \, D(\varepsilon) \, d\varepsilon$ AT $T = 0$

RECALL:

$$D(\varepsilon) = \frac{3}{2} \frac{N}{\varepsilon_F} \sqrt{\frac{\varepsilon}{\varepsilon_F}}$$ (DENSITY OF STATES)

$$U = \frac{3}{2} \frac{N}{\varepsilon_F^{3/2}} \int_0^{\varepsilon_F} \varepsilon^{3/2} \, d\varepsilon$$

SO

$$\boxed{U = \tfrac{3}{5} N \varepsilon_F}$$

(2) $P\,dV = T\,dS - dU$

(A) SO AT $T=0$

$$P = -\left.\frac{\partial U}{\partial V}\right|_S$$

NOW FROM PROB. 1

$$U = \tfrac{3}{5} N \varepsilon_F \quad \text{AND} \quad \varepsilon_F \sim n^{2/3}$$

$$\text{SO} \quad U \sim \left(\frac{1}{V}\right)^{2/3} \quad \text{SAY} \quad U = K_1 \left(\frac{1}{V}\right)^{2/3}$$

(K_1 ← CONSTANT)

$$\therefore \quad P = -\left(K_1 \left(\frac{1}{V}\right)^{+5/3}\right)\left(-\frac{2}{3}\right)$$

$$P = +\frac{2}{3} K_1 \left(\frac{1}{V}\right)^{2/3}\left(\frac{1}{V}\right)$$

$$\boxed{P = \tfrac{2}{3}\, U_0/V}$$

(THIS COULD BE DERIVED FROM KINETIC THEORY, AND HOLDS FOR A CLASSICAL, FERMI & BOSE GAS)

(B) SINCE $B = -V \frac{\partial P}{\partial V}$,

FROM PART (A),

$$P \sim K_2 \left(\frac{1}{V}\right)^{5/3}$$

↑ CONSTANT

SO

$$\boxed{B = \tfrac{5}{3} P = \frac{10}{9} \frac{U_0}{V}}$$

(C) $$B = \frac{10}{9} \frac{\left(\frac{3}{5} N \varepsilon_F\right)}{V} = \frac{2}{3} n \varepsilon_F$$

FROM TABLE 6.1

$$B = \frac{2}{3}\left(\frac{1.40 \times 10^{22}}{cm^3}\right)\left(2.12\ eV\right) \times \left(1.6 \times 10^{-12} \frac{erg}{eV}\right)$$

$$\boxed{B = 3.2 \times 10^{10} \frac{erg}{cm^3} = 3.2 \times 10^{10} \frac{DYNE}{cm^2}}$$

(FROM TABLE 3.3 (EXPERIMENT) $B = 3.2 \times 10^{10}$ DYNE/cm² !!

(5) THE GRAM MOLECULAR WEIGHT OF He^3 IS 3g/MOLE.
THUS THE "NUMBER DENSITY" FOR He^3 IS:

$$n = \frac{0.081\,g}{cm^3} \times \frac{1\,MOLE}{3g} \times \frac{6.02 \times 10^{23}}{MOLE} = \frac{1.63 \times 10^{22}}{cm^3}$$

NOW:

$$\mathcal{E}_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

$$= \underbrace{\left(\frac{\hbar^2}{2m_e a_0^2}\right)}_{13.6\,eV} \underbrace{\frac{m_e}{m_{He^3}}}_{\frac{1}{3(1836)}} \left(3\pi^2 n\right)^{2/3} \underbrace{a_0^2}_{(0.53 \times 10^{-8}\,cm)}$$

$$\mathcal{E}_F = 4.27 \times 10^{-4}\,eV \left(\approx 6.8 \times 10^{-16}\,ERG \quad 6.8 \times 10^{-23}\,JOULE\right)$$

$$T_F = \mathcal{E}_F / k_B = 5.0\,K$$