HW # 10

(3) (A).

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

(MAXWELL EQUATION IN A MEDIUM)
(i.e. AMPERE'S LAW WITH DISPLACEMENT CURRENT)

TAKE $\nabla \times$ OF BOTH SIDES
USE

$$\nabla \times \nabla \times \vec{B} = -\nabla^2 \vec{B}$$

$$= \frac{4\pi}{c}\left(\nabla \times \vec{J} + \frac{\partial}{\partial t}\nabla \times \vec{E}\right)$$

NOW $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{c\,\partial t}$

$$\nabla \times \vec{J} = \nabla \times \vec{J}_N + \nabla \times \vec{J}_S$$

$$= \sigma_0 \nabla \times \vec{E} - \frac{c}{4\pi \lambda_L^2}\underbrace{\nabla \times \vec{A}}_{\vec{B}}$$

$\vec{J}_N = \sigma_0 \vec{E}$

SO $$-\nabla^2 \vec{B} + \frac{\partial^2 B}{c^2 \partial t^2} = -\frac{4\pi}{c}\left(\frac{\sigma_0 \partial \vec{B}}{c\,\partial t} + \frac{c}{4\pi\lambda_L^2}\vec{B}\right)$$

ASSUME $\vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$
(WHERE $\vec{k}$ CAN BE COMPLEX) $\Rightarrow$

$$-\frac{\omega^2}{c^2} + \vec{k}\cdot\vec{k} = \frac{4\pi\sigma_0(i\omega)}{c^2} - \frac{1}{\lambda_L^2}$$

$$\Rightarrow \boxed{k^2c^2 = \omega^2 + 4\pi i \sigma_0 \omega - \frac{c^2}{\lambda_L^2}}$$

$\lambda_L^2 = \left(\frac{mc^2}{4\pi n_s e^2}\right)$

(B) TAKE $\sigma_0 = \frac{n_N e^2 \tau}{m}$

FOR $\omega\tau \ll 1$ WE DROP THE SECOND TERM ON RIGHT

$$\Rightarrow k^2c^2 = \omega^2 - \frac{c^2}{\lambda_L^2}$$

AND NORMAL ELECTRONS DO NOT INFLUENCE THE DISPERSION.

NOW $\frac{c^2}{\lambda_L^2} = \omega_p^2$. SINCE $\omega^2 \ll \omega_p^2$
WE CAN ALSO DROP $\omega^2 \Rightarrow$

$$k^2c^2 = -c^2/\lambda_L^2 \Rightarrow k = i/\lambda_L$$

THAT IS, THERE IS NO PROPAGATING MODE INSIDE (i.e. $\vec{B}$ FIELD DIES OUT).

(5)

(A) $$\vec{J} = -\frac{c}{4\pi\lambda_L^2}\vec{A}$$

$$\frac{\partial \vec{J}}{\partial t} = -\frac{c}{4\pi\lambda_L^2}\frac{\partial \vec{A}}{\partial t} = \frac{c^2}{4\pi\lambda_L^2}\vec{E}$$

(B) LET $m\frac{d\vec{V}}{dt} = q\vec{E}$

THEN

$$\frac{\partial(nq\vec{V})}{dt} = \frac{m}{q}\frac{d\vec{V}}{dt}\frac{c^2}{4\pi\lambda_L^2}$$

($nq\vec{V} = \vec{J}$)

$$\Rightarrow \frac{nq^2}{m}\frac{4\pi\lambda_L^2}{c^2} = 1 \Rightarrow \boxed{\lambda_L^2 = \frac{mc^2}{4\pi nq^2}}$$