Chapter 32 - Magnetism of Matter; Maxwell's Equations

1. The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist.

2. (a) The orbital magnetic moment is given by

$$\vec{\mu}_{orb} = \frac{q}{2m}\vec{L}_{orb}.$$

   For macroscopic orbits, the angular momentum is $L = mvr = m\omega r^2$. (It also possible to get this result from $\mu = NiA$ and considering the current to be $i = q/T = q\omega/2\pi$.)

   (b) The spin angular momentum of an electron is quantized and has the z component

$$\mu_{s,z} = \pm\frac{eh}{4\pi m}$$

   where $h = 6.63 \times 10^{-34}$ Js is Planck's constant.

3. Magnetic materials.

   (a) Diamagnetism is exhibited by all common materials but is so feeble that it can be masked. A diamagnetic material placed in an external magnetic field develops a magnetic dipole moment directed opposite to the external field. When the external field is removed, the dipole moment disappears. If the external field is nonuniform, the diamagnetic material is repelled from a region of greater magnetic field toward a region of lesser field.

   (b) Paramagnetism is due to the interaction of magnetic dipoles in the material. A paramagnetic material placed in an external magnetic field develops a magnetic dipole moment in the direction of the external field. If the field is removed, the induced dipole moment will disappear. If the field is nonuniform, the paramagnetic material is attracted toward a region of greater magnetic field from a region of lesser field.

   (c) Ferromagnetism is the property responsible for what is commonly called "magnetic" materials. A ferromagnetic material placed in an external magnetic field develops a strong magnetic dipole moment in the direction of the external field. If the field is removed, the ferromagnetic material can remained magnetized. If the field is nonuniform, the ferromagnetic material is attracted toward a region of greater magnetic field from a region of lesser field. A ferromagnetic material has similar properties to a paramagnetic material. The strength of the interactions is much greater in ferromagnetic materials.

4. Maxwell adds

$$\oint \vec{B} \cdot d\vec{s} = \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

   to Ampere's law and creates the Ampere-Maxwell law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{encl} + \mu_0\epsilon_0\frac{d\Phi_E}{dt}.$$

5. We identify the effective current from the changing electric flux as the displacement current

$$i_d = \epsilon_0 \frac{d\Phi_E}{dt}.$$

When a current is charging a parallel plate capacitor, it can be shown that displacement current in the gap of the capacitor is equal to the charging current.

6. Maxwell's equations summarize electricity and magnetism.

   (a) Gauss' law for electricity

   $$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$

   relates net electric flux to net enclosed electric charge. It says that electric fields come from electric charges.

   (b) Gauss' law of magnetism

   $$\oint \vec{B} \cdot d\vec{A} = 0$$

   relates net magnetic flux to net enclosed magnetic charge. It says that magnetic monopoles do not exist.

   (c) Faraday's law

   $$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

   relates induced electric field to changing magnetic flux. It says that a changing magnetic flux creates an electric field.

   (d) Ampere-Maxwell law

   $$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

   relates induced magnetic field to current and changing electric flux. It says that magnetic fields are created by moving electric charges or changing electric flux.

Chapter 33 - Electromagnetic Oscillations and Alternating Current

1. An LC circuit oscillates with a characteristic angular frequency $\omega = 1/\sqrt{LC}$.

2. The charge on the capacitor varies as

$$q(t) = Q_0 \cos(\omega t + \phi)$$

and the current is

$$i(t) = -\omega Q_0 \sin(\omega t + \phi) = -I_0 \sin(\omega t + \phi).$$

If we connect the capacitor to a potential V, its charge is $Q_0 = CV$. Often we take $\phi = 0$ or $\phi = \pi/2$ and we deal with $\cos \omega t$ or $\sin \omega t$.

3. The energy in the capacitor is

$$U_E(t) = \frac{q^2}{2C} = \frac{Q_0^2}{2C}\cos^2(\omega t + \phi)$$

and the energy in the inductor is

$$U_B(t) = \frac{1}{2}Li^2 = \frac{L}{2}I_0^2\sin^2(\omega t + \phi) = \frac{Q_0^2}{2C}\sin^2(\omega t + \phi).$$

Note that:

(a) The maximum values of U_E and U_B are both $Q_0^2/2C$. Energy is exchanged between the capacitor and inductor.

(b) At any instant the sum of U_E and U_B is equal to $Q_0^2/2C$, a constant. Energy is conserved.

(c) When U_E is a maximum, U_B is zero, and conversely. At some instant of time all the energy is either in the capacitor or in the inductor.

4. An ideal transformer is an iron core on which are wound a primary coil of N_P turns and a secondary coil of N_S turns. If the primary coil is connected across an alternating current generator, the primary and secondary voltages and currents are related by

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}.$$

Chapter 34 - Electromagnetic Waves

1. The electric field and the magnetic field satisfy a wave equation. We can express the plane wave solutions as

$$\begin{aligned} E(x,t) &= E_0\sin(kx - \omega t) \\ B(x,t) &= B_0\sin(kx - \omega t). \end{aligned}$$

The fields are related by $B = E/c$.

2. The electromagnetic wave is a transverse wave that propagates in the direction of $\vec{E} \times \vec{B}$. The speed of the wave is $c = 1/\sqrt{\mu_0\epsilon_0} = \omega/k = \lambda f$. The wave number is $k = 2\pi/\lambda$ and the angular frequency is $\omega = 2\pi f$.

3. The energy density in the electric field is

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 E_0^2\sin^2(kx - \omega t)$$

and the energy density in the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}B_0^2\sin^2(kx - \omega t).$$

It can be shown that the energy is equally divided between the electric and magnetic fields. Therefore $u_E = u_B$ and we have that the total energy density of the electromagnetic wave is

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0} = \epsilon_0 E^2 = \frac{B^2}{\mu_0}.$$

4. The rate of energy transport per unit area is the Poynting vector. The quantity

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

gives the instantaneous rate of energy flow.

5. The time average of the Poynting vector is the intensity

$$I = \frac{E_0^2}{2c\mu_0} = \frac{E_{rms}^2}{c\mu_0}$$

The root mean square value $E_{rms} = E_0/\sqrt{2}$. (To find the value of B_0, find E_0 and use $B_0 = E_0/c$.) The intensity falls off from an isolated point source emitting power P as

$$I = \frac{P}{4\pi r^2}.$$

6. The electromagnetic wave also carries momentum.

   (a) If the light is totally absorbed, the magnitude of the momentum change is $\Delta p = \Delta U/c$. The radiation pressure (force per unit area) is

   $$p_r = \frac{I}{c}.$$

   (b) If the light is totally reflected, the magnitude of the momentum change is $\Delta p = 2\Delta U/c$. The radiation pressure is

   $$p_r = \frac{2I}{c}.$$

   Be sure to differentiate between the two ps. The first is momentum and the second is pressure.

7. The law of reflection says that the angle of incidence is equal to the angle of reflection ($\theta_{inc} = \theta_{refl}$).

8. The law of refraction is Snell's law. It states

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

The index of refraction (n) is a constant for the medium that the light travels through.

(a) If n_2 is equal to n_1, then θ_2 is equal to θ_1. In this case, refraction does not bend the light beam, which continues in an undeflected direction.

(b) If n_2 is greater than n_1, then θ_2 is less than θ_1. In this case, refraction bends the light beam away from the undeflected direction and towards the normal.

(c) If n_2 is less than n_1, then θ_2 is greater than θ_1. In this case, refraction bends the light beam away from the undeflected direction and away from the normal.

9. A wave encountering a boundary across which the index of refraction decreases will experience total internal reflection if the angle of incidence exceeds a critical angle θ_c where

$$\theta_c = \sin^{-1} \frac{n_2}{n_1}.$$