Chapter 25 - Electric Potential

1. The work done by the electric field is equal to the negative of the change in the electric potential energy
$$W_{AB} = -(U_B - U_A).$$

2. The work done by an external force (i.e., other than the electric force) is equal to the sum of the change in the kinetic energy and the change in the potential energy
$$W_{AB} = (K_B - K_A) + (U_B - U_A).$$
Usually the only force acting is the force due to the electric field. Then $W_{AB} = 0$ and we have conservation of energy:
$$\begin{aligned} 0 &= (K_B - K_A) + (U_B - U_A) \\ K_A + U_A &= K_B + U_B. \end{aligned}$$
A charge wants to travel from high to low potential energy.

3. The potential difference (ΔV) experienced by a point charge q traveling from A to B is defined as
$$\begin{aligned} \Delta V = V_B - V_A &= -\frac{W_{AB}}{q} \\ \Delta V &= \frac{\Delta U}{q}. \end{aligned}$$
   (a) The electric potential is a scalar and can be positive, negative, or zero. (Compare to the electric field.)
   (b) Since only potential difference is defined, any convenient location can be chosen as the zero of potential. Usually it is taken at infinity.

4. The potential due to a point charge is
$$V = \frac{Kq}{r}$$

5. The potential can be calculated from the superposition of point charges.
   (a) Above the center of a ring of charge, $V = \frac{KQ}{\sqrt{R^2+z^2}}$.
   (b) Above the center of a disk, $V = \frac{\sigma}{2\epsilon_0}(\sqrt{z^2+R^2} - z)$.
   (c) Above the edge of a line of charge, $V = \frac{KQ}{L}\ln(\frac{L+\sqrt{L^2+d^2}}{d})$.

6. The potential difference can be calculated from the electric field
$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}.$$

(a) Electric field lines point from high to low electric potential.

(b) A positive charge wants to travel from high to low potential.

(c) A negative charge wants to travel from low to high potential.

(d) Equipotential surfaces are perpendicular to electric field lines.

(e) The potential of an isolated conductor is constant and is equal the potential at the surface of the conductor.

7. The electric field can be calculated from the electric potential.

$$E_x = -\frac{\partial V}{\partial x} \text{ and } E_y = -\frac{\partial V}{\partial y}.$$

8. The electric potential energy of a system of charges is the total work needed to assemble the system charges from infinity.

$$U = \sum_i \frac{1}{2} Q_i V_i.$$

It is important to realize that V_i is the potential experienced by Q_i and <u>not the potential due to Q_i</u>. Another formulation involves considering all the pairs of charges present and adding the energy due to each pair. For three charges (and it can be generalized to more charges), we have

$$U = \frac{Kq_1q_2}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}}.$$

Chapter 26 - Capacitance

1. A capacitor consists of two isolated conductors (the plates) with equal and opposite charges $+q$ and $-q$. Its capacitance C is defined as

$$C = \frac{q}{V}$$

where V is the potential difference between the plates.

2. Capacitance is a geometrical factor and depends only of the shape and dimensions involved in the capacitor.

(a) Parallel plate capacitors $C = \frac{\epsilon_0 A}{d}$.

(b) Spherical capacitors $C = \frac{ab}{K(b-a)}$.

(c) Cylindrical capacitors $C = \frac{L}{2K\ln(b/a)}$.

(d) An isolated sphere $C = R/K$.

3. Capacitors in parallel have the same potential. The total charge for the system is the sum of the charges on the individual capacitors. The equivalent capacitance is

$$C_P = C_1 + C_2 + C_3 + \ldots$$

4. Capacitors in series have the same charge. The total potential for the system is the sum of the potentials for the individual capacitors. The equivalent capacitance is given by
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

5. The work done in charging the capacitor is stored in the electric field of the capacitor. The energy stored in the capacitor is
$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV.$$
The energy density (energy per unit volume) is
$$u_E = \frac{1}{2}\epsilon_0 E^2$$

6. Some "experimental" facts
   (a) If a capacitor is charged by a battery and the capacitance is changed while the battery is still connected, the potential across the capacitor is unchanged. The charge, however, changes.
   (b) If a capacitor is charged by a battery and the capacitance is changed after the battery is disconnected, the charge on the capacitor is unchanged. The potential changes.
   (c) Inserting a dielectric (insulator) increases the capacitance of a capacitor.

Chapter 27 - Current and Resistance

1. The current is the rate of charge flow
$$I = \frac{dQ}{dt}.$$

2. *Conventional* current flows is the direction that a positive charge would flow.

3. The current density (J) is defined as the current per unit area
$$J = \frac{I}{A}.$$
If J depends on the position then the differential current is used
$$dI = J\,dA.$$

4. The current is proportional to the potential difference (Ohm's law):
$$V = IR.$$
The current flows from high to low potential.

5. Resistance is a property of an object and depends on the material used and its geometry. Resistivity is a property of a material. They are related through

$$R = \frac{\rho L}{A}.$$

6. The rate of energy transfer, or power, in an electrical device is the potential difference across the device times the current through the device:

$$P = IV.$$

Chapter 28 - Circuits

1. The emf device (battery) moves current from low to high potential.
2. Kirchhoff's Laws
   (a) The algebraic sum of the changes in potential encountered in a complete transversal of any loop of a circuit must be zero. (Loop rule.)
   (b) The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction. (Junction rule.)
3. Resistors in series have the same current. The total potential across the system is equal to the sum of the potentials across each resistor. The equivalent resistance is given by:

$$R_S = R_1 + R_2 + R_3 + \ldots$$

4. Resistors in parallel have the same potential. The total current through the system is equal to the sum of the currents through each resistor. The equivalent resistance is given by

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

5. There is a finite time needed to charge a capacitor. The time constant is

$$\tau = RC.$$

   (a) When charging an uncharged capacitor, the charge on the capacitor starts out at zero and rises:

$$Q = C\mathcal{E}(1 - e^{-t/\tau})$$

   and the current starts large and drops off to zero:

$$I = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

(b) When discharging a charged capacitor, the charge on the capacitor starts out large and also drops off to zero:

$$Q = C\mathcal{E}e^{-t/\tau}$$

and the current starts out large and drops off to zero:

$$I = -\frac{\mathcal{E}}{R}e^{-t/\tau}.$$

The minus sign indicates that the current in the discharging circuit is in the opposite direction to the current in the charging circuit.