Math Trivia

Cylindrical coordinates

$$x = \rho \cos\phi \qquad y = \rho \sin\phi$$

$$d\ell^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2, \qquad d^3v = \rho\, d\rho\, d\phi\, dz$$

$$\hat{\boldsymbol{\rho}} = \cos\phi\, \hat{\boldsymbol{x}} + \sin\phi\, \hat{\boldsymbol{y}}, \qquad \hat{\boldsymbol{\phi}} = -\sin\phi\, \hat{\boldsymbol{x}} + \cos\phi\, \hat{\boldsymbol{y}}$$

$$\nabla^2\Phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\Big(\rho\frac{\partial\Phi}{\partial\rho}\Big) + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Spherical coordinates

$$x = r\sin\theta\cos\phi \qquad y = r\sin\theta\sin\phi \qquad z = r\cos\theta$$

$$d\ell^2 = dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2, \qquad d^3v = r^2\, dr\ \sin\theta\, d\theta\, d\phi$$

$$\hat{\boldsymbol{r}} = \sin\theta\cos\phi\, \hat{\boldsymbol{x}} + \sin\theta\sin\phi\, \hat{\boldsymbol{y}} + \cos\theta\, \hat{\boldsymbol{z}}$$

$$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\, \hat{\boldsymbol{x}} + \cos\theta\sin\phi\, \hat{\boldsymbol{y}} - \sin\theta\, \hat{\boldsymbol{z}}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\Big(r^2\frac{\partial\Phi}{\partial r}\Big) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\Big(\sin\theta\,\frac{\partial\Phi}{\partial\theta}\Big) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

Gamma function

$$\Gamma(x) = \int_0^\infty t^{x-1}\, dt\, e^{-t} \qquad \Gamma(x+1) = x\,\Gamma(x) \qquad \Gamma(n) = (n-1)! \qquad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

Beta function

$$B(p,q) = \int_0^1 dt\, t^{p-1}\,(1-t)^{q-1} = 2\int_0^{\pi/2} d\theta\ \sin^{2p-1}\theta\ \cos^{2q-1}\theta = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}$$

Gaussian integral

$$\int_{-\infty}^\infty dx\ \exp\Big(-\frac{x^2}{2a^2}\Big) = \sqrt{2\pi}\,|a| \qquad \int_{-\infty}^\infty \frac{x^{2n}\,dx}{\sqrt{2\pi a^2}}\exp\Big(-\frac{x^2}{2a^2}\Big) = (2n-1)!!\, a^{2n}$$

Lorentzian (Cauchy) integral

$$\int_{-\infty}^\infty \frac{dx}{x^2+a^2} = \frac{\pi}{|a|} \qquad \int_{-\infty}^\infty \frac{dx}{(x^2+a^2)^p} = \frac{1}{|a|^{2p-1}}\,\frac{\sqrt{\pi}\,\Gamma(p-\frac{1}{2})}{\Gamma(p)}$$

Kronecker delta

$$\delta_{ij} = \hat{\boldsymbol{x}}_i\cdot\hat{\boldsymbol{x}}_j = \begin{cases} 1 & i=j \\ 0 & i\neq j\end{cases} \qquad \delta_{ij} = \delta_{ji} \qquad \delta_{ij}x_j = x_i \text{ (implicit summation)}$$

Levi-Civita (permutation) tensor

$$\epsilon_{ijk} = \begin{cases} +1 & ijk = 123,\ 231,\ 312 \\ -1 & ijk = 132,\ 213,\ 321 \\ \ \ 0 & \text{otherwise}\end{cases} \qquad \epsilon_{ijk} = -\epsilon_{jik} \text{ (all pairs)}$$

$$\epsilon_{ijk} = \hat{\boldsymbol{x}}_i\cdot(\hat{\boldsymbol{x}}_j\times\hat{\boldsymbol{x}}_k) \qquad \delta_{ij}\epsilon_{ijk} = 0 \qquad \epsilon_{ijk}\epsilon_{kmn} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$$