Some Series Expansions

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n$$

$$\int_0^x \frac{dx'}{1-x'} = -\ln(1-x) = x + \frac{1}{2}\,x^2 + \frac{1}{3}\,x^3 + \frac{1}{4}\,x^4 + \cdots = \sum_{n=1}^{\infty} \frac{1}{n}\,x^n$$

$$\ln(1+x) = x - \frac{1}{2}\,x^2 + \frac{1}{3}\,x^3 - \frac{1}{4}\,x^4 + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\,x^n$$

$$x\,\frac{d}{dx}\,\frac{1}{(1-x)} = \frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + 4x^4 + \cdots = \sum_{n=1}^{\infty} n\,x^n$$

$$(1+x)^p = 1 + p\,x + \frac{1}{2}\,p(p-1)\,x^2 + \frac{1}{6}\,p(p-1)(p-2)\,x^3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}\,\frac{p!}{(p-n)!}\,x^n$$

$$e^x = 1 + x + \frac{1}{2}\,x^2 + \frac{1}{6}\,x^3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}\,x^n$$

$$\sin x = x - \frac{1}{6}x^3 + \frac{1}{120}\,x^5 - \frac{1}{5040}\,x^7 + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}\,x^{2k+1}$$

$$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}\,x^4 - \frac{1}{720}\,x^6 + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\,x^{2k}$$