12/4/2017 Physics 3D, 4d. → PDE.

Separation of variables.

$$\boxed{\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = -j}$$ wave (source).

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V(x)\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

Both: $\boxed{\psi = \Psi(\vec{x})e^{-i\omega t}}$

(time translation invariant).

$$\nabla^2\psi - \frac{1}{c^2}(-i\omega)^2\psi = \nabla^2\psi + \frac{\omega^2}{c^2}\psi = -j$$

$\frac{\omega}{c} = k$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = \hbar\omega\psi$$

$\hbar\omega = \frac{\hbar^2k^2}{2m}$

$$(\nabla^2 + k^2)\psi = -j$$ (21.52) (21.53) (p.737)

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = \frac{\hbar^2k^2}{2m}\psi \rightarrow (\nabla^2 + k^2)\psi = \frac{2mV\psi}{\hbar^2}$$

[unnumbered. p.741.]

Helmholtz equation

$$\boxed{(\nabla^2 + k^2)\psi = -j}$$

$e^{im\phi}$

$Y_{lm}(\theta,\phi)$

$$\psi = \psi_0 + \text{"}(\nabla^2+k^2)^{-1}\text{"}(-j)$$

$$\boxed{(\nabla^2+k^2)\psi_0 = 0} \quad (\text{B. cond.}).$$

$$(\nabla^2+k^2)\,G = -\delta^{(3)}(\vec{x})$$

$$\boxed{G(\vec{x}-\vec{x}') = \frac{1}{4\pi}\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}}$$

HW #6

$$(\nabla^2+k^2)\frac{e^{ikr}}{r} = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)\frac{e^{ikr}}{r} + k^2\frac{e^{ikr}}{r}$$

$$= \left(\nabla^2\frac{1}{r}\right)e^{ikr} + 2\left(\vec{\nabla}\frac{1}{r}\right)\cdot\left(\vec{\nabla}e^{ikr}\right) + \frac{1}{r}\left(\nabla^2 e^{ikr}\right) + k^2\frac{e^{ikr}}{r}$$

$$-4\pi\delta(\vec{r})\cdot e^{ikr}$$

$$= \cancel{\frac{e^{ikr}}{r}} + 2\left(-\frac{\hat{r}}{r^2}\right)\cdot\left(ik\hat{r}\,e^{ikr}\right)$$

$$+ \frac{1}{r}\left((ik)^2 + \frac{2}{r}(ik)\right)e^{ikr} + k^2\frac{e^{ikr}}{r}$$

$$\psi(\vec{x}) = \psi_0(\vec{x}) + \int \frac{d^3x'}{4\pi} \frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} j(\vec{x}')$$

$$\nabla^2 + k^2 \to (0) + \int \frac{d^3x'}{\cancel{4\pi}} \left(-\delta(\vec{x}-\vec{x}')\right) j(\vec{x}') = -j(\vec{x})$$ ✓

<u>wave optics</u>:

$$\psi(\vec{x}) = \psi_0 + \int \frac{d^3x'}{4\pi} j(\vec{x}') \frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$$

$x' < d$ $\qquad$ $r \gg d$

$$|\vec{x}-\vec{x}'| = \sqrt{(\vec{x}-\vec{x}')\cdot(\vec{x}-\vec{x}')} = \sqrt{r^2 + r'^2 - 2\vec{x}\cdot\vec{x}'}$$

$$= r\sqrt{1 + \frac{r'^2}{r^2} - \frac{2\vec{x}\cdot\vec{x}'}{r^2}} \simeq r\left(1 - \frac{1}{2}\cdot\frac{2\vec{x}\cdot\vec{x}'}{r^2}\right)$$

$$= r - \left(\frac{\vec{x}}{r}\right)\cdot\vec{x}' = r - \hat{r}\cdot\vec{x}'$$

$$\psi(\vec{x}) = \underbrace{\psi_0}_{\text{incoming plane wave}} + \underbrace{\frac{e^{ikr}}{r}\int d^3x'\, j(\vec{x}')\, e^{-ik\hat{r}\cdot\vec{x}'}}_{\text{outgoing spherical wave}}$$

Then:

$$\psi(\vec{x}) = \psi_0 - \int \frac{d^3x'}{4\pi} \frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} \frac{2m}{\hbar^2} V(\vec{x}') \psi(\vec{x}')$$

$r \gg d$

$$\psi_0 - \frac{e^{ikr}}{r} \int d^3x' \left(\frac{m}{2\pi\hbar^2}\right) V(\vec{x}') \psi(\vec{x}') e^{-ik\hat{r}\cdot\vec{x}'}$$

scattering $\rightarrow$ outgoing spherical waves

Born approximation $\rightarrow$ $\psi(\vec{x}) \simeq \psi_0(\vec{x})$ inside integral

problem 17.15 p. 576

(6)

$$\boxed{\frac{dy}{dx} = y} \qquad y(0) = 1$$

guess: $y = 1$. $\frac{dy}{dx} = 0$ X.

Integrate. $\int_0^x \left(\frac{dy}{dx'}\right) dx' = y(x) - y(0) = \int_0^x y(x') \, dx'$

$$\boxed{y(x) = 1 + \int_0^x y(x') \, dx'}$$

guess: $y_0 = 0$ $\quad y_1(x) = 1 + \cancel{\int_0^x y_0(x') \, dx'} = 1$

$y_1 = 1$ $\quad y_2(x) = 1 + \int_0^x y_1 \, dx' = 1 + x.$

$y_2 = 1 + x$ $\quad y_3(x) = 1 + \int_0^x (1 + x') \, dx' = 1 + x + \frac{1}{2}x^2$

$\boxed{y_3 = 1 + x + \frac{1}{2}x^2}$ $\quad y_4(x) = 1 + \int_0^x \left(1 + x' + \frac{1}{2}x'^2\right) dx'$

$$= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

converges to $\sum_{n=0}^{\infty} \frac{1}{n!} x^n = e^x$ ✓