12/1/2017 linear operators → matrices, eigenvalues
diff. eq. → orthogonal functions

Fourier series . Fourier transforms
parseval. theorem.
δ-function

Uses: 3d. $F(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \tilde{F}(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$

$\nabla^2 G = -\delta \;\rightarrow\; \tilde{G} = \frac{1}{k^2}$

wave equation . separation of variables

$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = (\;.\;)\;(\circ)$

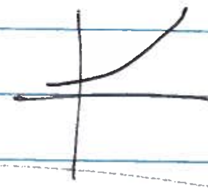
$\psi(x,y,z,t) = X(x)\,Y(y)\,Z(z)\,T(t)$

$\nabla^2\psi = X''\cdot YZT + X\cdot Y''\cdot ZT + XYZ''\cdot T$
$+ XYZT''$

$\frac{\nabla^2\psi}{\psi} = \frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} - \frac{1}{c^2}\frac{T''}{T}$

$\frac{X''}{X}, \frac{Y''}{Y}, \frac{Z''}{Z}, \frac{T''}{T}$ each constant (why?).

positive $X'' = +\alpha^2 X$.



exponential $e^{\pm \alpha x}$ (sinh/cosh)

All Negative $-k_x^2 - k_y^2 - k_z^2 + \frac{\omega^2}{c^2} = 0$

$$\psi = \int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{2\pi}\, \tilde{\psi}(\vec{k},\omega)\cdot e^{i\vec{k}\cdot\vec{x}}\, e^{-i\omega t}$$

$\Box^2\psi = 0$

plane waves. = modes of operator $\Box^2$.

Schrödinger.

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(x)\psi(x) = i\hbar\frac{\partial\psi}{\partial t}$$

$T' = -i\omega T$

$\psi(\vec{x},t) = \psi(\vec{x})\,T(t)$. $T = e^{-i\omega t}$

$$i\hbar\frac{\partial T}{\partial t} = \hbar\omega T = ET.$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(x)\psi(x) = E\psi(x)$$

stationary state = fixed energy.

4d P.D.E. $\longrightarrow$ 3.D. eigenvalue problem.

3

$\nabla^2 \phi$ $\begin{pmatrix} x, y, z \\ \rho, \phi, t \\ r, \theta, \phi \end{pmatrix}$

cylindrical
spherical
prolate/oblate spheroidal
ellipsoidal
hyperbolic
paraboloidal

$$ds^2 = A^2 \cdot da^2 + B^2 \cdot db^2 + C^2 \cdot dc^2$$

$$g^2 = A^2 B^2 C^2$$

$$\nabla^2 F = \frac{1}{g} \cdot \left[\frac{\partial}{\partial a}\left(\frac{g}{A^2}\frac{\partial F}{\partial a}\right) + \frac{\partial}{\partial b}\left(\frac{g}{B^2}\frac{\partial F}{\partial b}\right) + \frac{\partial}{\partial c}\left(\frac{g}{C^2}\frac{\partial F}{\partial c}\right) \right]$$

Spherical. $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$

$$g^2 = r^4 \cdot \sin^2\theta \qquad \boxed{g = r^2 \sin\theta}$$

$$\nabla^2 F = \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r}\left(r^2 \sin\theta \frac{\partial F}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\frac{r^2 \sin\theta}{r^2} \frac{\partial F}{\partial \theta}\right)$$

$$+ \frac{1}{r^2 \sin\theta} \cdot \frac{\partial}{\partial \phi} \cdot \left(\frac{r^2 \sin\theta}{r^2 \sin^2\theta} \frac{\partial F}{\partial \phi}\right)$$

imp

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial F}{\partial r}\right) + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial F}{\partial \theta}\right)$$

$$+ \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 F}{\partial \phi^2}$$

legendre

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) = + l(l+1) R$

each term "self-adjoint" form.

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) = r^2 \lambda R \qquad \rho = r^2 \quad p = r^2.$$

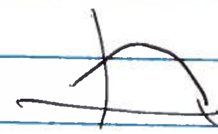
$$\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right) + q(x)\,y(x) + \lambda\rho\, y(x) = 0.$$

$$\int dx \cdot \frac{d}{dx}(py)\cdot y' = \left(y'\frac{d}{dx}(py)\right)\Big|_{\text{surface}} - \int dx\, p\frac{dy}{dx}\frac{dy'}{dx}$$

$p \neq 0$ on surface $\longleftrightarrow$ $y\big|_{\text{surface}} = 0$

$\Rightarrow$ negative feedback

$y'' = -(\ )\,y$



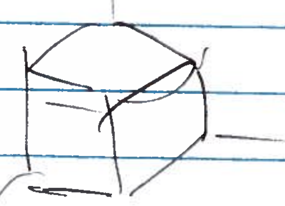
$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = 0$$

can't have.
$-k_x^2,\ -k_y^2,\ -k_z^2$

$\rho = 0$ on cube $\quad \nabla^2\Phi = 0 \quad$ + BC. $\Phi\big|_{top} = V(x,y)$

$0<x<a \quad 0<y<b \quad 0<z<c$

$$\Phi = \sin\alpha x \cdot \sin\beta y \cdot \sinh\gamma z.$$



$$\Phi\big|_{x=0} = \Phi(y=0) = \Phi(z=0) = 0$$

$\alpha = \frac{n\pi}{a} \quad \beta = \frac{m\pi}{b} \quad \Phi\big|_{x=a} \quad \Phi\big|_{y=b} = 0.$

$$\Phi\big|_{z=c} = \sum_{mn} A_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\cdot \sinh\gamma_{mn}c$$

$$= V_s(x,y).$$