

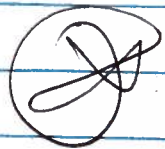
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Table 17.1

Polynomials

Chebyshev . $\int_{-1}^1 \frac{dx}{\sqrt{1-x^2}} T_n(x) T_m(x) = \begin{cases} \frac{\pi}{2} \delta_{nm} & (n \neq 0) \\ \pi & (n = m = 0) \end{cases}$

Laguerre . $\int_0^{\infty} dx e^{-x} L_n(x) L_m(x) = \delta_{nm}$



Hermite . $\int_{-\infty}^{\infty} dx e^{-x^2} H_n(x) H_m(x) = \frac{2^n n! \sqrt{\pi}}{\delta_{nm}}$

Legendre . $\int_{-1}^1 dx P_n(x) P_m(x) = \frac{2}{2n+1} \delta_{nm}$

Strange facts

$$H_{n+1} = 2xH_n - 2nH_{n-1}$$

$$\cos(n+1)x = \dots$$

$$H_0 = 1$$

$$H_1 = 2x$$

(n=0)

$$H_1 = 2xH_0 - (0)(H_{n-1})$$

$$(n=1) \quad H_2 = 2xH_1 - 2 \cdot 1 \cdot H_0 = 2x(2x) - 2 = 4x^2 - 2.$$

$$(n=2) \quad H_3 = 2xH_2 - 2 \cdot 2 \cdot H_1 = 2x(4x^2 - 2) - 4 \cdot 2x \\ = 8x^3 - 4x - 8x = \underline{8x^3 - 12x} \quad \checkmark$$

code

$$\frac{dH_n}{dx} = 2nH_{n-1}$$

$$\frac{\partial}{\partial x} (\cos nx) = -n \sin nx$$

" H^k " $\rightarrow H_k(x)$.

$$H_n(x+iy) = (H + 2iy)^k$$

$$H_2(x+iy) = H_2(x) + H_1(x) \cdot 2y \cdot 2 + 4y^2 \\ = (4x^2 - 2) + 8xy + 4y^2$$

$$4(x+iy)^2 - 2 = 4x^2 + 8xy + 4y^2 - 2 \quad \checkmark$$

Write:
$$S_{2n} = \sum_{n=1}^{\infty} C_n H_n(x).$$

$$\underline{n_1 = 2\sqrt{4}} \quad C_1 = \frac{1}{2e^{1/4}} \quad H_1 = 2x \quad C_1 H_1 = \frac{x}{e^{1/4}} = 0.7188x$$

$$\underline{n_3 = 48\sqrt{4}} \quad C_3 = \frac{1}{48e^{1/4}} \quad H_3 = 8x^3 - 12x$$

$$S_3 = \frac{5}{4e^{1/4}} x - \frac{1}{6e^{1/4}} x^3$$

$$= \underline{0.97350x} - 0.12980x^3$$

$$S_5 = \frac{41}{32e^{1/4}} x - \frac{5}{24e^{1/4}} x^3 + \frac{1}{120e^{1/4}} x^5$$

$$\underline{0.997834} \quad 0.162250 \quad 0.0064900$$

$$S_7 = \frac{493}{384e^{1/4}} x - \frac{41}{192e^{1/4}} x^3 + \frac{1}{96e^{1/4}} x^5 - \frac{1}{5040e^{1/4}} x^7$$

$$\underline{0.999867} \quad 0.166306 \quad 0.008112 \quad 0.000154.$$