11/29/2017

Table 17.1 Polynomials

Chebyshev: $\int_{-1}^{1} \frac{dx}{\sqrt{1-x^2}} T_m(x) T_n(x) = \frac{\pi}{2}\delta_{mn} \quad (m \neq 0)$

$\left(\pi \quad m=n=0 \right)$

Laguerre: $\int_{0}^{\infty} dx\, e^{-x} L_m(x) L_n(x) = \delta_{mn}$

Hermite: $\int_{-\infty}^{\infty} dx\, e^{-x^2} H_m(x) H_n(x) = \delta_{mn}\, 2^n n! \sqrt{\pi}$

Legendre: $\int_{-1}^{1} dx\, P_m(x) P_n(x) = \frac{2}{2m+1} \delta_{mn}$

Strange facts.

$\cos(n+1)x = \dots$

$$H_{n+1} = 2x H_n - 2n H_{n-1}$$

$H_0 = 1$ $H_1 = 2x$

$(n=0)$ $H_1 = 2x H_0 - (0)(H_{n-1})$

$(n=1)$ $H_2 = 2x H_1 - 2\cdot 1\cdot H_0 = 2x(2x) - 2 = 4x^2 - 2.$

$(n=2)$ $H_3 = 2x H_2 - 2\cdot 2\cdot H_1 = 2x(4x^2-2) - 4\cdot 2x$

$= 8x^3 - 4x - 8x = 8x^3 - 12x$ ✓

(code).

$$\frac{dH_n}{dx} = 2n H_{n-1}$$

$\frac{d}{dx}(\cos nx) = -n \sin nx$

"H^k" $\to H_k(x)$.

$$H_n(x+y) = (H + 2y)^k$$

$H_2(x+y) = H_2(x) + H_1(x)\cdot 2y\cdot 2 + 4y^2$

$= (4x^2-2) + 8xy + 4y^2$

$4(x+y)^2 - 2 = 4x^2 + 8xy + 4y^2 - 2$ ✓

Write. $\sin x = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} c_n H_n(x)$.

$m_1 = 2\sqrt{\pi}$ $\quad c_1 = \frac{1}{2e^{1/4}}$ $\quad H_1 = 2x$ $\quad c_1 H_1 = \frac{x}{e^{1/4}} = 0.7788x$

$m_3 = 48\sqrt{\pi}$ $\quad c_3 = \frac{1}{48e^{1/4}}$ $\quad H_3 = 8x^3 - 12x$

$$S_3 = \frac{5}{4e^{1/4}} x - \frac{1}{6e^{1/4}} x^3$$

$$= 0.97350\, x - 0.12980\, x^3$$

$$S_5 = \frac{41}{32e^{1/4}} x - \frac{5}{24e^{1/4}} x^3 + \frac{1}{120\, e^{1/4}} x^5$$

0.997839 $\quad$ 0.162250 $\quad$ 0.0064900

$$S_7 = \frac{493}{384e^{1/4}} x - \frac{41}{192e^{1/4}} x^3 + \frac{1}{96e^{1/4}} x^5 - \frac{1}{5040e^{1/4}} x^7$$

0.999867 $\quad$ 0.166306 $\quad$ 0.008112 $\quad$ 0.000154.