11/20/2017

$$\boxed{\begin{aligned} f(t) &= \int \frac{d\omega}{2\pi} \tilde{f}(\omega)\, e^{i\omega t} \\ \tilde{f}(\omega) &= \int dt\, f(t)\, e^{-i\omega t} \end{aligned}}$$

Use to solve. $\vec{F} = m\frac{d^2\vec{x}}{dt^2} = -k\vec{x} - b\vec{v} + \vec{F}_{ext}$.

$b = \gamma m$
$k = m\omega_0^2$

$$m\frac{d^2x}{dt^2} + \gamma m\frac{dx}{dt} + m\omega_0^2 x = F_{ext}(t)$$

write. $x = \int \frac{d\omega}{2\pi} \tilde{x}(\omega)\, e^{i\omega t}$

$$F = \int \frac{d\omega}{2\pi} \tilde{F}(\omega)\, e^{i\omega t}$$

$$\int \frac{d\omega}{2\pi} \left[\left(-m\omega^2 + i\gamma m\omega + m\omega_0^2\right)\tilde{x}\right] e^{i\omega t} =$$

$$m\left(-\omega^2 + i\gamma\omega + \omega_0^2\right)\tilde{x} = \tilde{F}$$

$$\boxed{\tilde{x} = \frac{\tilde{F}/m}{(\omega_0^2 - \omega^2 + i\gamma\omega)}}$$

$$x(t) = \int \frac{d\omega}{2\pi} \frac{\tilde{F}/m}{(\omega_0^2 - \omega^2 + i\gamma\omega)} e^{i\omega t}$$

$$= \int \frac{d\omega}{2\pi} \frac{e^{i\omega t}}{\omega_0^2 - \omega^2 + i\gamma\omega} \cdot \int dt' \, F(t') e^{-i\omega t'}$$

$$= \int dt' \frac{F(t')}{m} \int \frac{d\omega}{2\pi} \frac{e^{i\omega(t-t')}}{\omega_0^2 - \omega^2 + i\gamma\omega}$$

can do this integral: contour.

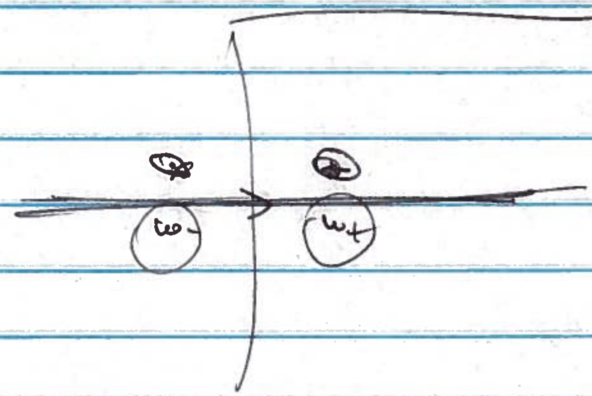
poles: $\omega^2 - i\gamma\omega - \omega_0^2 = 0$

$$\omega = \frac{i\gamma \pm \sqrt{(i\gamma)^2 + 4 \cdot 1 \cdot \omega_0^2}}{2}$$

$$= \frac{i\gamma}{2} \pm \sqrt{\omega_0^2 - \frac{\gamma^2}{4}} = \frac{i\gamma}{2} \pm \omega'$$

underdamped case

$$\omega'^2 = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$$



$$\omega_+ = +\omega' + i\gamma/2$$

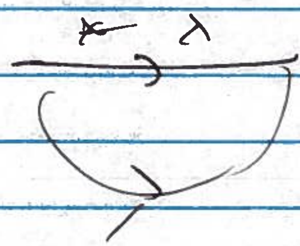
$$\omega_- = -\omega' + i\gamma/2$$

3

contour $t' > t$ $\quad e^{i\omega(t-t')} = e^{-i\omega(t'-t)}$

exponentially suppressed for $\boxed{\mathrm{Im}(\omega) < 0}$

$$e^{-i(\Omega - i\Gamma)(t'-t)} \Rightarrow e^{-i\Omega(t'-t)}\,\underbrace{e^{-\Gamma(t'-t)}}_{>0}$$

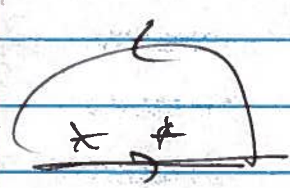


$$\oint = \int_{-\infty}^{\infty} d\omega = 0.$$

$t' < t$ $\quad e^{i\omega(t-t')}$ exponentially suppressed for $\boxed{\mathrm{Im}\,\omega > 0}$

$$e^{i(\Omega + i\Gamma)(t-t')} = e^{i\Omega(t-t') - \Gamma(t-t')}$$

$$e^{i\Omega(t-t')}\,e^{-\Gamma(t-t')}$$



$$\oint \frac{d\omega}{2\pi}\frac{e^{i\omega(t-t')}}{(-)(\omega-\omega_+)(\omega-\omega_-)}$$

$$= 2\pi i \cdot \frac{(-)}{2\pi}\left[\frac{e^{i\omega_+(t-t')}}{\omega_+ - \omega_-} + \frac{e^{i\omega_-(t-t')}}{\omega_- - \omega_+}\right]$$

$$= 2\pi i\,\frac{(-)}{2\pi}\left(\frac{e^{i\omega'(t-t')}}{2\omega'}\,e^{-\frac{\gamma}{2}(t-t')} + \frac{e^{-i\omega'(t-t')}}{-2\omega'}\,e^{-\frac{\gamma}{2}(t-t')}\right)$$

$$= \cancel{2\pi} i \cdot \frac{1}{\cancel{2\pi}} \cdot \frac{(-1)}{2\omega'} \cdot e^{-\frac{\gamma}{2}(t-t')} \left[e^{i\omega'(t-t')} - e^{-i\omega'(t-t')} \right]$$

$$= \frac{-i}{2\omega'} \, e^{-\frac{\gamma}{2}(t-t')} \cdot 2i \sin \omega'(t-t')$$

$$= \frac{1}{\omega'} \, e^{-\frac{\gamma}{2}(t-t')} \sin \omega'(t-t')$$

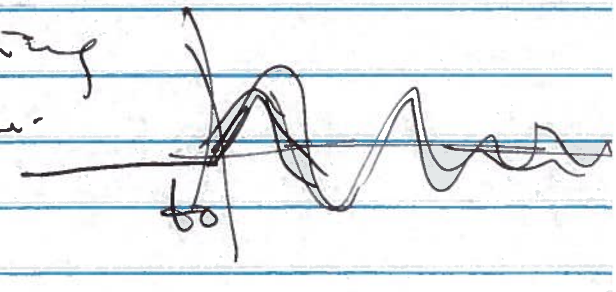
$$\boxed{x(t) = \int_{-\infty}^{t} dt' \cdot \frac{F(t')}{m} \frac{1}{\omega'} \, e^{-\frac{\gamma}{2}(t-t')} \sin \omega'(t-t')}$$

$[F] = [ma^2]$ $\quad \left[\frac{F}{m}\right] = \frac{[x]}{[t^2]}$

$$\boxed{F(t') = J \cdot \delta(t'-t_0)}$$ impulse. $\int F dt = J$ at $t = t_0$,

$$\boxed{x(t) = \frac{J}{m\omega'} \cdot e^{-\frac{\gamma}{2}(t-t_0)} \sin \omega'(t-t_0)}$$

natural response to being hit with hammer.



$$F = F_0 \cos \omega t$$

$$x(t) = \int_{-\infty}^{\infty} dt' \, \frac{F_0}{m\omega'} \cos \omega t' \, e^{-\frac{\gamma}{2}(t-t')} \sin \omega'(t-t')$$

$$\omega'^2 + \left(\tfrac{\gamma}{2}\right)^2 - \omega_0^2$$

$$= \frac{\cancel{\omega'} \left[\left(\omega'^2 - \omega^2 + (\gamma/2)^2\right) \cos \omega t + \gamma \omega \sin \omega t \right]}{\left((\omega - \omega')^2 + \left(\tfrac{\gamma}{2}\right)^2\right)\left((\omega + \omega')^2 + \left(\tfrac{\gamma}{2}\right)^2\right)} \frac{F_0}{m \cancel{\omega'}}$$

$$x(t) = \frac{(\omega_0^2 - \omega^2) \cos \omega t + \gamma \omega \sin \omega t}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

resonance!

$$\nabla^2 \Phi = -\rho$$ poisson's equation

$$\Phi = -\text{"}\frac{1}{\nabla^2}\text{"}\rho$$ [Green's function $= (\nabla^2)^{-1}$]

$$\nabla^2 G = -\delta^{(3)}(\vec{r}-\vec{r}\,')$$

$$\Phi = \int d^3r'\, G(\vec{r}-\vec{r}\,')\,\rho(\vec{r}\,')$$

$$\nabla^2\Phi = \int d^3r'\,(\nabla^2 G)\,\rho(r') = \int d^3r'\left(-\delta(\vec{r}-\vec{r}\,')\right)\rho(\vec{r}\,') = -\rho(\vec{r})$$

$(\nabla^2-\lambda^2)$

$$\cancel{\nabla}^2 G = -\delta^{(3)}(\vec{r}-\vec{r}\,') \quad \vec{R}$$

$$G = \int \frac{d^3k}{(2\pi)^3}\,\tilde{G}(\vec{k})\, e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')}$$

$$\nabla^2 G = \int \frac{d^3k}{(2\pi)^3}\,\tilde{G}(\vec{k})\,(-i\vec{k})\cdot(-i\vec{k})\, e^{i\vec{k}\cdot\vec{R}}$$

$$\delta(\vec{r}-\vec{r}\,') = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{R}}$$

$$\int \frac{d^3k}{(2\pi)^3}\left[+\tilde{G}\left(-k^2-\lambda^2\right)\right] e^{i\vec{k}\cdot\vec{R}}$$

$$= \int \frac{d^3k}{(2\pi)^3}\,(-)\, e^{i\vec{k}\cdot\vec{R}}$$

$$\boxed{\tilde{G} = \frac{1}{k^2+\lambda^2}}$$

$$G(\vec{x}-\vec{x}') = \int \frac{d^3k}{(2\pi)^3}\, \tilde{G}\, e^{i\vec{k}\cdot\vec{R}}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2+\lambda^2}\, e^{i\vec{k}\cdot\vec{R}}$$

$$\frac{1}{(2\pi)^3} \int \frac{k^2 dk}{k^2+\lambda^2} \cdot \underbrace{\int d\Omega\, e^{i\vec{k}\cdot\vec{R}}}$$

$$\int d\mu \cdot d\phi\, e^{ikR\mu} = 2\pi \int_{-1}^{1} e^{ikR\mu} d\mu$$

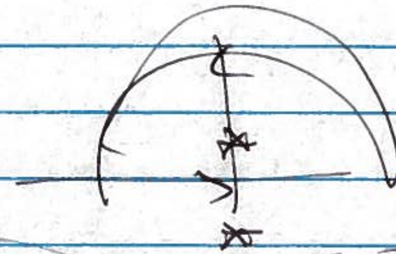
$$= 4\pi \cdot \frac{\sin kR}{kR}$$

$$= \frac{4\pi}{(2\pi)^3} \int_0^\infty \frac{k^{\cancel{2}} dk}{k^2+\lambda^2} \frac{\sin kR}{\cancel{k}R}$$

$$= \frac{1}{2\pi^2} \cdot \frac{1}{R} \int_0^\infty \frac{k\,dk\, \sin kR}{k^2+\lambda^2}$$

$$= \frac{1}{4\pi^2} \frac{1}{R}\, \mathrm{Im} \int_{-\infty}^{\infty} \frac{k\,dk\, e^{ikR}}{k^2+\lambda^2}$$

$R > 0 \Rightarrow \mathrm{Im}\, k > 0$



poles: $k^2 + \lambda^2 = 0$ $\quad k = \pm i\lambda \qquad k = +i\lambda$

$$G = \mathrm{Im}\left\{ 2\pi i \cdot \frac{1}{4\pi^2 R} \left(\frac{k e^{ikR}}{k + i\lambda} \right)\Bigg|_{k = i\lambda} \right\}$$

$$= \mathrm{Im}\left\{ \frac{\cancel{2\pi} i}{4\pi^2 R} \cdot \frac{\cancel{i\lambda} e^{-\lambda R}}{\cancel{2i\lambda}} \right\}$$

$$G = \frac{1}{4\pi} \cdot \frac{1}{R} e^{-\lambda R} \qquad \lambda \to 0: \quad G = \frac{1}{4\pi R}$$

$$\nabla^2 \Phi = -\rho/\varepsilon_0$$

$$\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int d^3 r' \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}$$