11/8/2017. Vector.

$$\boxed{\vec{V} = V_x \hat{x} + V_y \hat{y} + V_z \hat{z}}$$ vector.

$$\boxed{V_x = \hat{x} \cdot \vec{V}}$$ components.

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left(a_n \cdot \cos nx + b_n \sin nx \right)$$

basis vectors

components

$$\int_{-\pi}^{\pi} f(x)\, dx = \int_{-\pi}^{\pi} \left[\frac{1}{2} a_0 + \sum a_n \cos nx + b_n \sin nx \right] dx$$

$$\boxed{\langle \cos nx \rangle = \langle \sin nx \rangle \to 0}$$

$$= \frac{1}{2} a_0 \cdot 2\pi \quad \boxed{a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx}$$

$$\int_{-\pi}^{\pi} f(x) \cos px\, dx = \int_{-\pi}^{\pi} \left(\frac{1}{2} a_0 + \sum_n a_n \cos nx + b_n \sin nx \right) \cos px\, dx$$

$$\boxed{\langle \cos mx \cos nx \rangle = \frac{1}{2} \delta_{mn}}$$

$$= a_p \cdot \frac{1}{2} \cdot 2\pi$$

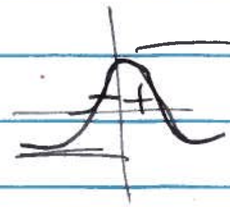
$$\boxed{a_p = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos px\, dx}$$

$$\boxed{b_p = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin px\, dx}$$

Example: $f(x) = \begin{cases} +1 & 0<x<\pi \\ -1 & -\pi<x<0 . \end{cases}$

$\int_{-\pi}^{\pi} f(x)\,dx =$  $= 0 .$

odd function $\langle f \rangle = 0$

$\int_{-\pi}^{\pi} f(x) \cos px\,dx =$  $= 0 .$

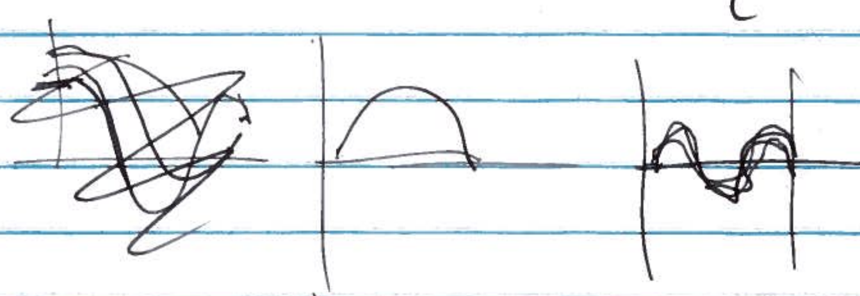
$(x<0)$ cancels $(x>0)$

$$\int_{-\pi}^{\pi} f(x) \sin px = \int_{-\pi}^{\pi} \operatorname{sgn}(x) \sin px\,dx$$

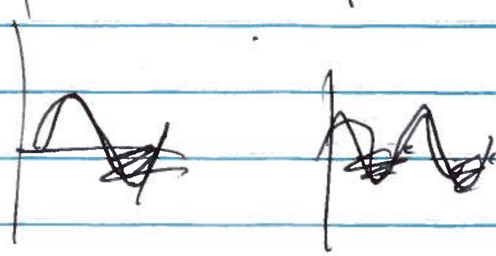
$$= 2\int_0^{\pi} \sin px\,dx$$

$$= 2\left[-\frac{1}{p}\cos px\right]_0^{\pi}$$

$$= \frac{2}{p}\left(1-(-1)^p\right) = \begin{cases} \frac{4}{p} & p \text{ odd} \\ 0 & p \text{ even} . \end{cases}$$



odd.



even

$$\int_{-\pi}^{\pi} f(x)\sin px\,dx = \frac{4}{p} \quad (p\ \text{odd}) = \frac{b_p}{\pi}$$

$$\boxed{\text{series fn} = \sum_{p\ \text{odd}} \frac{4}{\pi p}\sin px}$$

$$-\ln(1-z) = z + \tfrac{1}{2}z^2 + \cdots = \sum_n \tfrac{1}{n} z^n$$

$$\ln(1+z) = z - \tfrac{1}{2}z^2 + \cdots = \sum_n \tfrac{1}{n}(-1)^n z^n$$

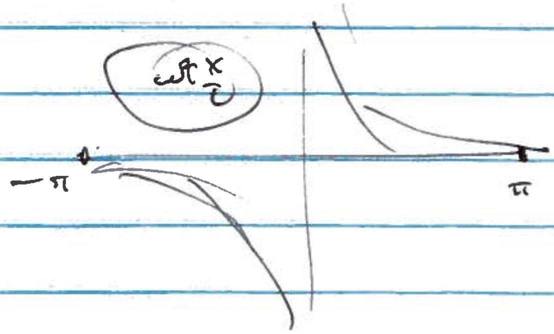
$$\sum_{n\ \text{odd}} \tfrac{1}{n} z^n = \tfrac{1}{2}\ln\left(\frac{1+z}{1-z}\right) \qquad z = e^{ix}$$

$$\text{series} \rightarrow \mathrm{Im}\left[\frac{4}{\pi}\cdot\frac{1}{2}\ln\left(\frac{1+e^{ix}}{1-e^{ix}}\right)\right]$$

$$\frac{(1+e^{ix})}{(1-e^{ix})}\cdot\frac{(1-e^{-ix})}{(1-e^{-ix})} = \frac{1+e^{ix}-e^{-ix}-1}{1-e^{ix}-e^{-ix}+1}$$

$$= \frac{2i\sin x}{2-2\cos x} = \frac{4i\sin\frac{x}{2}\cos\frac{x}{2}}{4\sin^2\frac{x}{2}} = i\cot\frac{x}{2}$$

$$\mathrm{Im}(\ln) = \text{phase} = \begin{cases} +\pi/2 & (\cot\frac{x}{2} > 0) \\ -\pi/2 & (\cot\frac{x}{2} < 0) \end{cases}$$



Finite Sums $f_N(x) = \frac{4}{\pi} \sum_{\substack{n=1 \\ odd}}^{N} \frac{1}{n} \sin nx$

$$f_N\left(\frac{(N-1)\pi}{N}\right) = \frac{4}{\pi} \sum_{\substack{n=1 \\ odd}}^{N} \frac{1}{n} \sin\left[n\left(\pi - \frac{\pi}{N}\right)\right]$$

$$\cancel{\sin n\pi \cos \frac{n\pi}{N}} - \cos n\pi \sin \frac{n\pi}{N}$$

$$= \frac{4}{\pi} \sum_{\substack{n=1 \\ odd}}^{N} \frac{1}{n} \sin \frac{n\pi}{N} \frac{(\pi/N)}{(\pi/N)}$$

$$= \frac{2}{\pi} \sum_{n=1}^{N} \frac{\sin\left(\frac{n\pi}{N}\right)}{\left(\frac{n\pi}{N}\right)} \left(\frac{2\pi}{N}\right)$$

Let $y = \frac{n\pi}{N}$

$\Delta y = 2 \cdot \frac{\pi}{N}$ (odds)

$$= \frac{2}{\pi} \sum_{\substack{n=1 \\ odd}}^{N} \frac{\sin y_n}{y_n} \Delta y \rightarrow \frac{2}{\pi} \int_0^\pi \frac{\sin y}{y} dy$$

Sin Integral $(\pi) = 1.85193705 \ldots$ $(N \rightarrow \frac{\pi}{2})$

$\times \frac{2}{\pi}$ overshoot $\rightarrow 1.1789797444 \ldots$

"Gibbs overshoot"

(5)

$$\int_{-\pi}^{\pi} f(x) \sin nx \, dx = 0 \qquad b_n = 0$$

$$\int_{-\pi}^{\pi} f(x) \cdot \cos nx \, dx = 2\int_0^{\pi} \frac{x}{\pi} \cos nx \, dx.$$

$$= \frac{2}{\pi} \cdot \left\{ \frac{1}{n^2} \left(\cancel{n\pi \sin n\pi} + \underbrace{\cos n\pi}_{(-1)^n} - 1 \right) \right.$$

$$= \begin{cases} \frac{4}{\pi^2 n^2}(-) & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$\frac{d}{dx}$ = square wave $\cdot \frac{1}{\pi}$

$$\text{triangle wave} = \frac{1}{2} - \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{4}{\pi^2 n^2} \cos nx$$

$x = \pi$. $$1 = \frac{1}{2} - \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{4}{\pi^2 n^2} \cancel{(-1)^n} (-1)$$

$$\cancel{\sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{(-1)^{n+1}}{n^2}} \qquad \boxed{\sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{4} \cdot \left(1 - \frac{1}{2}\right) = \frac{\pi^2}{8}}$$

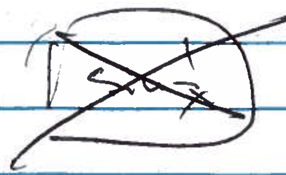
$$\left(1 - \frac{1}{2^p}\right) \zeta(p) \to \left(1 - \frac{1}{4}\right)\left(\frac{\pi^2}{6}\right)$$

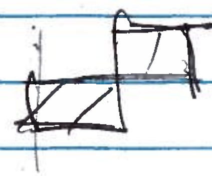
HW 1 contour integral

Triangle wave doesn't overshoot.

Dirichlet conditions:

$f(x)$ single-valued

finite # min/max 

finite # discontinuities 

$$\int_0^{2\pi} |f(x)|\, dx = \int_{-\pi}^{\pi} |f(x)|\, dx < \infty.$$

$\Rightarrow$ series converges where f continuous

$\Rightarrow$  $\leftarrow$ (midpoint) at jumps.

$\Leftarrow$ $\boxed{\sin nx = 0}$ at $x = 0, \pi$