10/23/2017.

Linear Algebra Chapters 7, 8

(know?)

scalar . a

vector $\vec{V} = V_i = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ (column).

matrix $\overleftrightarrow{A} = A_{ij}$

$$\overleftrightarrow{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{pmatrix}$$

$M \times N$.

i: 1st index = row

j = 2nd index = column.

All shapes = "vector space"

$A + B = (A+B)_{ij} = (A_{ij}) + (B_{ij})$ (same shape). (8.29)

aA $\quad (aA)_{ij} = a(A_{ij})$

AB $\quad (AB)_{ij} = \sum_{k=1}^{N} A_{ik} B_{kj}$ $\boxed{\S 8.4.2}$

$$\begin{pmatrix} * & * & * \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} * & \cdot \\ * & \cdot \\ * & \cdot \end{pmatrix}$$

dot product of row of A with column of B

$(M \times N) \cdot (N \times P) \Rightarrow (M \times P)$

Non commutative $AB \neq BA$ in general (8.35)

$P \neq M$, can't even do : different spaces

$P = M$ $\quad (M \times N)\cdot(N \times M) \to (M \times M)$

$(N \times M)\cdot(M \times N) \to (N \times N)$

$A(BC) = (AB)C$ Associative (8.34)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 1+6+15 & 2+8+18 \\ 4+15+30 & 8+20+36 \end{pmatrix} = \begin{pmatrix} 22 & 28 \\ 49 & 64 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ 5+24 & 10+30 & 15+36 \end{pmatrix} = \begin{pmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 29 & 40 & 51 \end{pmatrix}$$

To have $AB = BA$ requires at the least square $N \times N$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad AB = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix} \qquad 19+50 = 69$$

$$B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \qquad BA = \begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix} \qquad 23+46 = 69$$

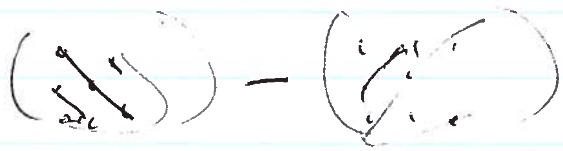
Square matrix:

determinant $= \sum_{\text{all permutations}}$ (one from each row/column)

$$\det A = \varepsilon_{ijk} A_{1i} A_{2j} A_{3k}$$ $i \neq j \neq k \neq i$ all different.

§8.9

$$= A_{11}A_{22}A_{33} + A_{12}A_{23}A_{31} + A_{13}A_{21}A_{32} - A_{13}A_{22}A_{31} - A_{11}A_{23}A_{32} - A_{12}A_{21}A_{33}$$



(8.46)
(8.47)

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

$\det A = 4 - 6 = -2$ $\det B = 40 - 42 = -2$

$AB = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$ $\det AB = 950 - 946 = 4$

$A + B = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$ $\det(A+B) = 72 - 80 = -8$

$\det AB = (\det A)(\det B)$

$\det(A+B) \neq (\det A) + (\det B)$ in general

4

Identity $\quad I \cdot A = AI = A$

§4.3

$$I_{ij} = \delta_{ij}$$

$$(IA)_{ij} = (I_{ik})(A_{kj}) = \delta_{ik} A_{kj} = A_{ij}$$

$$\det I = \varepsilon_{ijk}\, \delta_{1i}\, \delta_{2j}\, \delta_{3k} = \varepsilon_{123} = 1$$

If (exists): $\quad A A^{-1} = A^{-1} A = I$

$$(\det A)(\det A^{-1}) = \det I) = 1.$$

$$\det(A^{-1}) = \frac{1}{(\det A)} = (\det A)^{-1}$$

Invertibility requires $\det A \neq 0$

$$B^{-1} A^{-1} A B = B^{-1} B = I$$

$$B^{-1} A^{-1} = (AB)^{-1}$$

group

5

Transpose: $A^T_{ij} = A_{ji}$ $M \times N \to N \times M$.

$(AB)^T_{ij} = (A_{ik}B_{kj})^T = A_{jk}B_{ki}$ $i \leftrightarrow j$

$= B^T_{ik} A^T_{kj} = (B^T A^T)_{ij}$

§8.6

$(AB)^T = B^T A^T$

Symmetric $A^T = A$

$\mathrm{Tr}\, A = A_{ii} = \sum_{i=1}^{k} A_{ii}$

$\mathrm{Tr}\, AB = \mathrm{Tr}(A_{ik}B_{kj}) = A_{ik}B_{ki}$

$= B_{ki}A_{ik} = \mathrm{Tr}(BA)$

$\mathrm{Tr}(ABC) = \mathrm{Tr}(BCA) = \mathrm{Tr}(CAB)$

wrap, cyclic

$M \times 1$, a_{i1} $i = 1 \ldots M$ M rows, 1 column

"column vector"

$1 \times N$ a_{1j} $j = 1 \ldots N$ N columns, 1 row

"row vector"