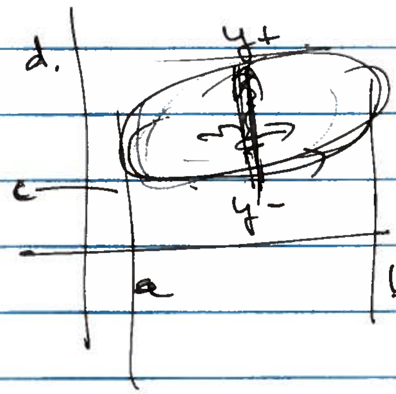
10 | 2 | 17

Theorems.

Fundamental Theorem of Calculus

$$\int_a^b \left[\frac{d}{dt} f(t)\right] dt = f(b) - f(a).$$

Do "this" in a plane.



$$\int_{\text{Area}} \frac{\partial P}{\partial y} \, dy \, dx = \int_a^b dx \int_{y-}^{y+} dy \, \frac{\partial P}{\partial y}$$

$$= \int_a^b dx \left[\underset{(\text{upper})}{P(x, y+)} - \underset{(\text{lower})}{P(x, y-)} \right]$$

$$= -\int_a^b \underset{\text{lower}}{P(x, y-)} \, dx + \int_a^b \underset{\text{upper}}{P(x, y+)} \, dx$$

$$= -\int_a^b P(x, y-) \, dx - \int_b^a P(x, y+) \, dx = -\oint P \, dx.$$

CCW

Do again, $Q(x,y)$, x-integral first

$$\int_A \frac{\partial Q}{\partial x}\, dx\, dy = \int_c^d dy \left[\int_{x_-}^{x_+} \frac{\partial Q}{\partial x}\, dx \right)$$

$$= \int_c^d dy \left(\underbrace{Q(x_+, y)}_{\text{right}} - \underbrace{Q(x_-, y)}_{\text{left}} \right)$$

$$= \int_c^d dy\, Q(x_+, y) + \int_d^c dy\, Q(x_-, y)$$

$$= + \oint_S Q\, dy.$$

$$\boxed{\iint_{\text{Area}} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\, dy = \oint_{\text{curve}} \left(P\, dx + Q\, dy \right)}$$

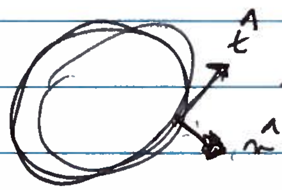
"Green's Theorem" (one of many).

(11.4)

$\vec{v} = v_x \hat{x} + v_y \hat{y}$ case $Q = v_x$, $P = -v_y$

$\vec{r} = x\hat{x} + y\hat{y}$

$P\,dx + Q\,dy = v_y\,dx - v_x\,dy$



$d\vec{r} = \hat{x}\,dx + \hat{y}\,dy = ds\,\hat{t}$

$$\boxed{\hat{n}\,ds = dy\,\hat{x} - dx\,\hat{y}}$$

$\hat{n}\cdot\hat{t} = 0$ $|\hat{n}| = 1$

$$P\,dx + Q\,dy = (v_y)(-dx) + (v_x)(dy)$$
$$= \vec{v}\cdot\hat{n}\,ds$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial v_x}{\partial x} - \left(-\frac{\partial v_y}{\partial y}\right) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = \operatorname{div}\vec{v}$$

$$\boxed{\iint_A \operatorname{div}\vec{v}\;\underbrace{dx\,dy}_{(dA)} = \oint_C \vec{v}\cdot\hat{n}\,ds}$$ 2D

$$\boxed{\iiint_V \operatorname{div}\vec{v}\,d^3V = \oint_S \vec{v}\cdot\hat{n}\,d^2S}$$ (11.18) $(\vec{v} \to \vec{a})$ $d\vec{S} = \hat{n}\,d^2S$

$Q = v_y$

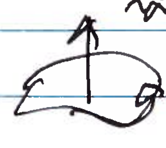
$P = v_x$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = (\text{curl}\,\vec{v})_z$$

$$P\,dx + Q\,dy = v_x\,dx + v_y\,dy = \vec{v}\cdot d\vec{r}$$

$$\boxed{\iint_A (\text{curl}\,\vec{v})\cdot\hat{n}\,\underbrace{dx\,dy}_{d^2A} = \oint_S \vec{v}\cdot d\vec{r}}$$

"Stokes Theorem"



$\hat{n}$, $d\vec{r}$ $\longleftrightarrow$ RHR

All follow from $\int_a^b \frac{\partial F}{\partial x}\,dx = F(b) - F(a)$.

All $\int_{\text{interior}} (\partial f) = \int_{\text{boundary}} (f)$

$$\boxed{\int_\Omega d\omega = \int_{\partial\Omega} \omega}$$

"generalized Stokes Theorem"

ω = differential form

d = exterior derivative

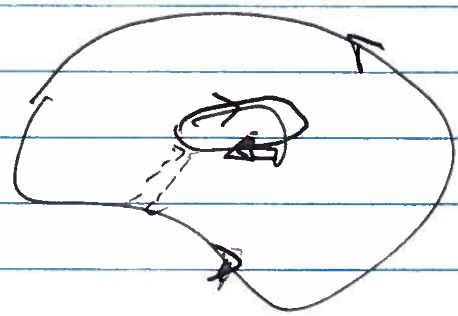
Ω = domain

∂ = Boundary.

$d^2 = 0$ $\rightarrow$ "$\partial^2 = 0$"

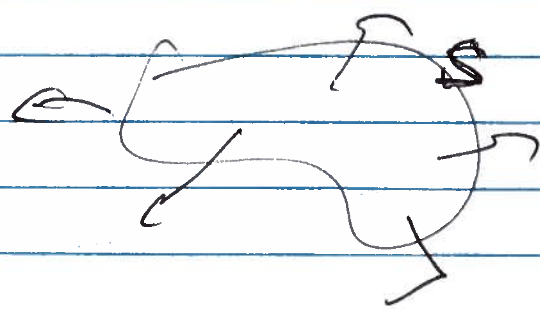
$\vec{\nabla}\times\vec{\nabla}f = 0$ $\quad$ $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

not "simply connected"



"extras" cancel.

$$\oint \vec{v}\cdot\hat{n}\, d^2a = \text{"Flux"} = \text{flow}$$



$\frac{dm}{dt}$ = mass flow through S

$$\delta m = \rho \cdot \Delta V = \rho \cdot A \cdot v_\perp \Delta t$$

$$\frac{dm}{dt} = -\oint_S (\rho\vec{v})\cdot\hat{n}\, d^2A$$

$$= \frac{d}{dt}\left(\int d^3v \cdot \rho\right) = \int_V d^3x \cdot \frac{\partial\rho}{\partial t}$$

$$\boxed{\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0}$$

§11.8.3

no numbers

6

Gauss's law

$$\oint_S \vec{E}\cdot\hat{n}\, d^2A = Q/\varepsilon_0 = \int_V d^3v\, \frac{\rho}{\varepsilon_0} = \int_V d^3v\, \vec{\nabla}\cdot\vec{E}$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0}$$

Outward flux $\Longleftrightarrow$ divergence

Ampère's law.

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I = \int_S \mu_0 \vec{J}\cdot\hat{n}\, d^2A = \int_S (\vec{\nabla}\times\vec{B})\cdot\hat{n}\, d^2a$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$$

circulation $\longleftrightarrow$ curl