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$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\vec{\nabla}\phi = \hat{x}\frac{\partial\phi}{\partial x} + \hat{y}\frac{\partial\phi}{\partial y} + \hat{z}\frac{\partial\phi}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\wedge\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\vec{\nabla}\times(\vec{\nabla}\phi) = 0$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$

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$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = \varepsilon_{ijk} \nabla_j (\vec{A} \times \vec{B})_k .$$

$$= \varepsilon_{ijk} \nabla_j \left(\varepsilon_{kmn} A_m B_n \right)$$

$$= \left(\delta_{im} \delta_{jn} - \delta_{in} \delta_{jm} \right) \left[(\nabla_j A_m) B_n + A_m (\nabla_j B_n) \right]$$

$$= B_j (\nabla_j A_i) - (\nabla_j A_j) B_i + A_i (\nabla_j B_j) - A_j (\nabla_j B_i)$$

$$= (\vec{B} \cdot \vec{\nabla}) \vec{A} - \vec{B} (\vec{\nabla} \cdot \vec{A}) + \vec{A} (\vec{\nabla} \cdot \vec{B}) - (\vec{A} \cdot \vec{\nabla}) \vec{B}$$

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Two terms in $\vec{A}$-direction, $\vec{B}$ contracted with $\vec{\nabla}$

Two terms in $\vec{B}$-direction, $\vec{A}$ contracted with $\vec{\nabla}$

$$\vec{\nabla} \times (\vec{a} \times \vec{x}) = \cancel{(\vec{x} \cdot \vec{\nabla}) \vec{a}} - \cancel{\vec{x} (\vec{\nabla} \cdot \vec{a})} + \vec{a} (\vec{\nabla} \cdot \vec{x}) - (\vec{a} \cdot \vec{\nabla}) \vec{x}$$

$$\vec{\nabla} \cdot \vec{x} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3 \quad (\text{dim.})$$

$$(\vec{a} \cdot \vec{\nabla}) \vec{x} = \left(a_x \frac{\partial}{\partial x} + a_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial z} \right) (x \hat{x} + y \hat{y} + z \hat{z})$$

$$= a_x \hat{x} + a_y \hat{y} + a_z \hat{z} = \vec{a}$$

$$= 3\vec{a} - \vec{a} = 2\vec{a}$$

$$\vec{A} = -y(x^2+y^2)\,\hat{x} + x(x^2+y^2)\,\hat{y}$$

$$\underline{\underline{\vec{\nabla}\cdot\vec{A}}} = -2xy + 2xy = \underline{\underline{0}}$$

!

$$\underline{\underline{\vec{\nabla}\times\vec{A}}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ -y(x^2+y^2) & x(x^2+y^2) & 0 \end{vmatrix} = \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= \hat{z}\left[\frac{\partial}{\partial x}\, x(x^2+y^2) - \frac{\partial}{\partial y}\left(-y(x^2+y^2)\right)\right]$$

$$= \hat{z}\left[3x^2+y^2+x^2+3y^2\right] = \underline{\underline{4(x^2+y^2)\,\hat{z}}}.$$

$$\underline{\underline{\vec{\nabla}\times(\vec{\nabla}\times\vec{A})}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & 4(x^2+y^2) \end{vmatrix}$$

$$= \hat{x}\,\frac{\partial}{\partial y}\left(4(x^2+y^2)\right) - \hat{y}\,\partial_x\left(4(x^2+y^2)\right) = \underline{\underline{8y\hat{x} - 8x\hat{y}}}$$

$$\underline{\underline{\nabla^2\vec{A}}} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left[-(yx^2+y^3)\hat{x} + (x^3+xy^2)\hat{y}\right]$$

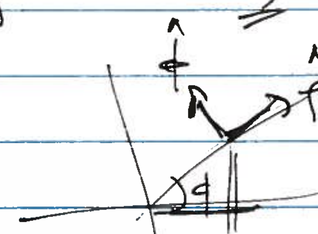
$$= \hat{x}\left[-(2y+6y)\right] + \hat{y}\left[6x+2x\right]$$

$$= \underline{\underline{-8y\hat{x} + 8x\hat{y}}}$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = 0 + \underline{\underline{8y\hat{x} - 8x\hat{y}}}$$

Polar coordinates (cylindrical). §10.9.1

(10.44) $x = \rho\cos\phi$, $y = \rho\sin\phi$ $\qquad \vec{x} = x\hat{x} + y\hat{y}$ $\qquad$ (z)



(10.46) $\hat{\rho} = \left(\frac{\partial \vec{x}}{\partial \rho}\right) = \hat{x}\cos\phi + \hat{y}\sin\phi$

(10.47) $\hat{\phi} = \frac{1}{\rho}\frac{\partial \vec{x}}{\partial \phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$

$$\boxed{\vec{\nabla} = \hat{\rho}\frac{\partial}{\partial\rho} + \frac{\hat{\phi}}{\rho}\frac{\partial}{\partial\phi}} \qquad \frac{\partial\hat{\rho}}{\partial\phi} = \hat{\phi} \qquad \frac{\partial\hat{\phi}}{\partial\phi} = -\hat{\rho}$$

$$d\vec{x} = \hat{\rho}\,d\rho + \rho\,d\phi\,\hat{\phi}$$

$$ds^2 = d\vec{x}\cdot d\vec{x} = d\rho^2 + \rho^2 d\phi^2$$ (p.354)

$$\vec{\nabla}\cdot\vec{A} = \left(\hat{\rho}\frac{\partial}{\partial\rho} + \frac{\hat{\phi}}{\rho}\frac{\partial}{\partial\phi}\right)\cdot\left(A_\rho\hat{\rho} + A_\phi\hat{\phi}\right)$$

$$= \hat{\rho}\cdot\left(\frac{\partial A_\rho}{\partial\rho}\hat{\rho} + \cancel{A_\rho\frac{\partial\hat{\rho}}{\partial\rho}} + \cancel{\frac{\partial A_\phi}{\partial\rho}\hat{\phi}} + \cancel{A_\phi\frac{\partial\hat{\phi}}{\partial\rho}}\right)$$

$$+ \frac{\hat{\phi}}{\rho}\cdot\left(\cancel{\frac{\partial A_\rho}{\partial\phi}\hat{\rho}} + A_\rho\frac{\partial\hat{\rho}}{\partial\phi} + \frac{\partial A_\phi}{\partial\phi}\hat{\phi} + \cancel{A_\phi\frac{\partial\hat{\phi}}{\partial\phi}}\right)$$

$$\boxed{\vec{\nabla}\cdot\vec{A} = \frac{\partial A_\rho}{\partial\rho} + \frac{1}{\rho}A_\rho + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi}}$$

Table 10.2