9/27/2017. $\hat{x}_i \cdot \hat{x}_j = \delta_{ij}$ orthonormal

$\hat{x}_i \times \hat{x}_j = \varepsilon_{ijk}\, \hat{x}_k$ RHR.

elephant x plum.

$\varepsilon_{ijk}\, \varepsilon_{kmn} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$

$\vec{A} \times (\vec{B} \times \vec{C}) = \underbrace{\vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})}_{\text{lies in } \vec{B}-\vec{C} \text{ plane}}$

$\perp (\hat{n}_{BC})$

product rules.

$$\frac{d}{dt}(a\vec{A}) = a \cdot \frac{d\vec{A}}{dt} + \left(\frac{da}{dt}\right)\vec{A}$$

$$\frac{d}{dt}(\vec{A} \cdot \vec{B}) = \vec{A} \cdot \left(\frac{d\vec{B}}{dt}\right) + \left(\frac{d\vec{A}}{dt}\right) \cdot \vec{B}$$

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{A} \times \left(\frac{d\vec{B}}{dt}\right) + \left(\frac{d\vec{A}}{dt}\right) \times \vec{B}$$

mechanics.

$\vec{x} = (x\hat{x} + y\hat{y} + z\hat{z})$ $\qquad \frac{d\hat{x}_i}{dt} = 0$

(NB: there are accelerating (rotating) coordinates.

$\frac{d\vec{x}}{dt} = (\dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z}) = \vec{v}$

constant speed $\qquad \frac{d v^2}{dt} = \frac{d}{dt}(\vec{v}\cdot\vec{v}) = 2\vec{v}\cdot\frac{d\vec{v}}{dt} = 0$

$\vec{v}\cdot\vec{a} = 0$

constant r. $\qquad \frac{d}{dt}(r^2) = 2\vec{r}\cdot\vec{v} = 0$

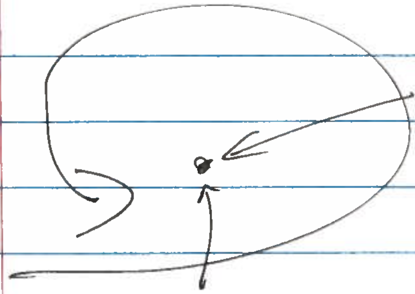
$\vec{r}\cdot\vec{v} = 0$

rotating: r, v both constant

$\frac{d}{dt}(\vec{r}\cdot\vec{v}) = \vec{r}\cdot\vec{a} + \vec{v}\cdot\vec{v} = 0$

$\vec{r}\cdot\vec{a} = -v^2$

$\vec{a} = -\frac{v^2}{r}\hat{r}$



$\vec F \propto \vec r$ "central force"

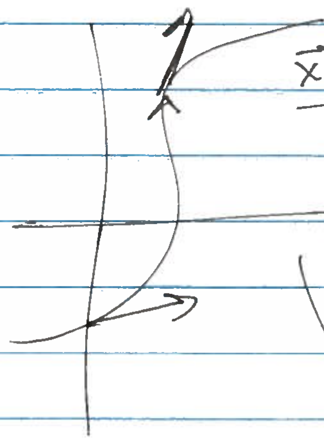
$\vec L = \vec r \times m\vec v$

$$\frac{d\vec L}{dt} = \cancel{\left(\frac{d\vec r}{dt}\right)\times m\vec v} + \vec r \times \left(m\frac{d\vec v}{dt}\right) = 0$$

$\vec v \times \vec v = 0$ $\vec r \times \vec r = 0$

$\frac{d\vec L}{dt} = 0$ $\vec L = \text{constant}$

Fields . $\phi(\vec{x},t)$, $\vec{A}(\vec{x},t)$ — scalar, vector-valued functions of space (time)



$\vec{x} = \vec{x}(s)$. parameterized curve (possibly straight line)

$\vec{u} = \frac{d\vec{x}}{ds}$ tangent at $\hat{x}(s)$.

$\phi = \phi(\vec{x}(s)) = \phi(s)$.

$$\frac{d\phi}{ds} = \frac{\partial\phi}{\partial x}\cdot\frac{dx}{ds} + \frac{\partial\phi}{\partial y}\cdot\frac{dy}{ds} + \frac{\partial\phi}{\partial z}\frac{dz}{ds}$$

$$= \left(\frac{\partial\phi}{\partial x_i}\right)\left(\frac{dx_i}{ds}\right) = \vec{\nabla}\phi\cdot\vec{u}$$

§10.7

$$\vec{\nabla}\phi = \left(\frac{\partial\phi}{\partial x_i}\hat{x}_i\right) = \hat{x}\frac{\partial\phi}{\partial x} + \hat{y}\frac{\partial\phi}{\partial y} + \hat{z}\frac{\partial\phi}{\partial z}$$ §10.7.1

"gradient of ϕ".

$S' = \{\phi = 0\}$ $\quad \frac{d\phi}{ds} = \vec{\nabla}\phi\cdot\vec{u} = 0 \quad (\phi \equiv 0)$

$\vec{x}$ in S' $\quad \vec{u} \neq 0$ (curve). $\quad \vec{\nabla}\phi \neq 0$ (non constant). $\rightarrow$ $\cos\theta = 0$

$\Longrightarrow$ $(\vec{\nabla}\phi)$ is $\perp$ surface $\{\phi = \text{constant}\}$

$\vec{\nabla}$ acting on vectors.

$$\vec{\nabla}\cdot\vec{A} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z\right)$$

$$= (\hat{x}\cdot\hat{x})\frac{\partial A_x}{\partial x} + \dots + (\hat{y}\cdot\hat{y})\frac{\partial A_y}{\partial y} + \dots + (\hat{z}\cdot\hat{z})\frac{\partial A_z}{\partial z}$$

$$\boxed{\vec{\nabla}\cdot\vec{A} = \text{div}\,\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

"divergence" (scalar) § 10.7.2

$$\vec{\nabla}\times\vec{A} = (\nabla_i\hat{x}_i)\times(A_j\hat{x}_j) = \varepsilon_{ijk}\nabla_i A_j\hat{x}_k.$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right).$$

= "curl $\vec{A}$" (vector) § 10.7.3

$\vec{\nabla}$ almost a vector.

(order).

Second derivatives.

$$\vec{\nabla}\cdot(\vec{\nabla}\phi) = \frac{\partial}{\partial x_i}\left(\frac{\partial\phi}{\partial x_i}\right) = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \nabla^2\phi.$$

"Laplacian"

$\nabla^2\phi = 0$ $\quad(\nabla^2\phi = \rho)$ Laplace (Poisson).

$\nabla^2\phi = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$. Wave.

$\nabla^2\psi = D\frac{\partial\phi}{\partial t}$ heat (diffusion.)

$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$. Schrödinger

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$\vec{\nabla}\times(\vec{\nabla}\phi) = 0$ $\quad \underbrace{\varepsilon_{ijk}}_{\text{odd}}\underbrace{\nabla_i\nabla_j}_{\text{even}}\phi = 0.$ $\quad d^2 = 0$

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = \quad \varepsilon_{ijk}\nabla_i\nabla_j A_k = 0.$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{C}) = \vec{\nabla}_A\times(\vec{\nabla}_B\times\vec{C})$$

$$= \vec{\nabla}_B(\vec{\nabla}_A\cdot\vec{C}) - (\vec{\nabla}_A\cdot\vec{\nabla}_B)\vec{C}$$

$$= \vec{\nabla}(\vec{\nabla}\cdot\vec{C}) - \nabla^2\vec{C} \qquad (10.41)$$

not the same thing