9/25/2017 Chapter 7. Vectors.

- Set, addition, scalar multiplication
- Arrow. length + direction
- Rotates like a vector
  ↳ preserves lengths and angles

(7.13) $|\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}}$ $\quad \vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta_{AB}$.

$\vec{A}' = R\vec{A}$ cm, Elm, Qm. (7.15)

$\cancel{\vec{A}\cdot\vec{B} =}$

$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ $\quad \hat{i}\ \hat{j}\ \hat{k}$

$= A_x\hat{e}_x + A_y\hat{e}_y + A_z\hat{e}_z$

$= A_i\hat{x}_i = A_i\hat{e}_i$ $\quad \sum_{i=1}^{3}$ implicit (Einstein)

$\vec{A}\cdot\vec{B} = (A_i\hat{x}_i)\cdot(B_j\hat{x}_j)$

$= A_iB_j(\hat{x}_i\cdot\hat{x}_j) = A_iB_j\delta_{ij} = A_iB_i$

$\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$ $\quad \begin{pmatrix} 1&0&0\\0&1&0\\0&0&1 \end{pmatrix}$ identity

2

$$(\hat{A} \pm \lambda \vec{B}) \cdot (\hat{A} \pm \lambda \vec{B}) \geq 0 .$$

$$= A^2 \pm 2\lambda \hat{A} \cdot \vec{B} + \lambda^2 B^2$$

true for $\lambda \cdot \frac{\partial}{\partial \lambda} = 0$ $\quad \pm 2\hat{A} \cdot \vec{B} + 2\lambda B^2 = 0$

$$\lambda = \pm \frac{\hat{A} \cdot \vec{B}}{B^2}$$

$$A^2 \mp 2\left(\frac{\hat{A} \cdot \vec{B}}{B^2}\right)(\hat{A} \cdot \vec{B}) + \left(\frac{\hat{A} \cdot \vec{B}}{B^2}\right)^2 B^2 \geq 0$$

$$\boxed{A^2 + B^2 \geq |\vec{A} \cdot \vec{B}|.}$$ $\cos\theta_{AB}$

$$\vec{A} \times \vec{B} = |\vec{A}| \, |\vec{B}| \sin\theta_{AB} \, \hat{n}_{AB}$$

$\hat{n}_{AB} \to$ RHR

1.33

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(A_y B_z - A_z B_y) + \hat{y}(A_z B_x - A_x B_z) + \hat{z}(A_x B_y - A_y B_x)$$

1.24

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$$\boxed{\vec{A} \times \vec{B} = \varepsilon_{ijk} \hat{x}_i A_j B_k}$$

$\vec{A} \times \vec{A} = 0$

1.20

$$\boxed{(\vec{A} \times \vec{B})_i = \varepsilon_{ijk} A_j B_k}$$

$$\boxed{(\vec{A} \cdot \vec{B})^2 + (\vec{A} \times \vec{B})^2 = A^2 B^2}$$ $\sin^2 + \cos^2 = 1$

$\varepsilon_{123} = 1$ $\quad \varepsilon_{ijk} = -\varepsilon_{jik}$ (all exchanges)

$\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij}$ $\quad \boxed{\hat{x}_i \times \hat{x}_j = \varepsilon_{ijk}\hat{x}_k}$

$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = +1$

$\varepsilon_{132} = \varepsilon_{213} = \varepsilon_{321} = -1$

Any two the same $\rightarrow$ 0. (21 $\begin{matrix} 112 & 113 \\ 221 & 223 \\ 331 & 332 \end{matrix}$ $\times 3$ + 111 $\times 3$)

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \varepsilon_{ijk} A_i B_j C_k$$

1.34

$$= \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$ cyclic

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$$\vec{A}\times(\vec{B}\times\vec{C}) = \varepsilon_{ijk}\hat{x}_i A_j (\varepsilon_{kmn} B_m C_n)$$

$$= \varepsilon_{ijk}\varepsilon_{kmn} A_j B_m C_n$$

$\sum_k$ one term $(k \neq i, k \neq j)$ $\quad m, n \neq k \Rightarrow$

mn also ij in some order.

$mn = ij \quad \varepsilon_{ijk}\varepsilon_{kij} = (\varepsilon_{ijk})^2 = (\pm 1)^2 = +1.$

$m \neq n$.

$mn = ji \quad \varepsilon_{ijk}\varepsilon_{kji} = \varepsilon_{ijk}\varepsilon_{jik} = (+1)(-1) = -1$

$$\varepsilon_{ijk}\,\varepsilon_{kmn} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm})\,\hat{x}_i A_j B_m C_n$$

$$= \hat{x}_i A_j B_i C_j - \hat{x}_i A_j B_j C_i$$

$$= \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\boxed{\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})}$$

$$(\vec{A}\times\vec{B})\times\vec{C} = -\vec{C}\times(\vec{A}\times\vec{B})$$

$$= +\vec{C}\times(\vec{B}\times\vec{A}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{A}(\vec{B}\cdot\vec{C})$$

Not the same. Cross product is not associative

Need parens

$$\vec{A}\times(\vec{B}\times\vec{C}) + \vec{B}\times(\vec{C}\times\vec{A}) + \vec{C}\times(\vec{A}\times\vec{B}) = 0$$

"Jacobi identity"