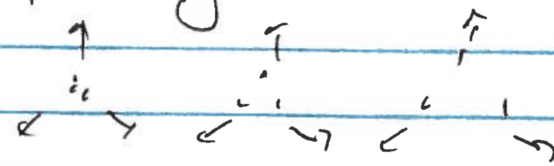
9/6/2017

Partial derivatives. $f(r_i, t)$ $\quad \frac{\partial f}{\partial r_i}, \frac{\partial f}{\partial t}$

changing variables.

Expanding universe. "proper" "comoving"



$$\boxed{\vec{r} = a(t)\,\vec{x}}$$

$$\underbrace{\dot{r} = \dot{a}x}_{\text{"Hubble."}} + a\dot{x} \leftarrow \text{"peculiar"}$$

$$v = \frac{\dot{a}}{a} r = (v = Hr)$$

$$df = \left(\frac{\partial f}{\partial t}\right)_r dt + \left(\frac{\partial f}{\partial r}\right)_t dr$$

$r = r(t,x)$ $\quad dr = \left(\frac{\partial r}{\partial t}\right)_x dt + \left(\frac{\partial r}{\partial x}\right)_t dx$.

$$df = \left(\frac{\partial f}{\partial t}\right)_r dt + \left(\frac{\partial f}{\partial r}\right)_t \left[\left(\frac{\partial r}{\partial t}\right)_x dt + \left(\frac{\partial r}{\partial x}\right)_t dx\right]$$

$$= \left[\left(\frac{\partial f}{\partial t}\right)_r + \left(\frac{\partial f}{\partial r}\right)_t \left(\frac{\partial r}{\partial t}\right)_x\right] dt$$

$$+ \left(\frac{\partial f}{\partial r}\right)_t \left(\frac{\partial r}{\partial x}\right)_t dx.$$

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$$df = \left(\frac{\partial f}{\partial t}\right)_x dt + \left(\frac{\partial f}{\partial x}\right)_t dx$$

$$\boxed{\left(\frac{\partial f}{\partial t}\right)_x = \left(\frac{\partial f}{\partial t}\right)_r + \left(\frac{\partial f}{\partial r}\right)_t \left(\frac{\partial r}{\partial t}\right)_x}$$

$\left(\frac{d}{a}\right) r = a x$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}$$ "convective derivative"

$$\left(\frac{\partial f}{\partial x}\right)_t = \left(\frac{\partial f}{\partial r}\right)_t \left(\frac{\partial r}{\partial x}\right)_t$$ "chain rule":

$\oint \vec{S} \cdot \vec{S}$

$f = f(r(x), t)$

$$\frac{\partial f}{\partial x} = \frac{f(r(x+\Delta x), t) - f(r(x), t)}{\Delta x}$$

$\Delta r =$

$$= \frac{f\left(r + \frac{\partial r}{\partial x}\Delta x\right) - f(r)}{\delta x} = \frac{f(r) + \frac{\partial f}{\partial r}\cdot\left(\frac{\partial r}{\partial x}\Delta x\right) - f(r)}{\Delta x}$$

$$= \left(\frac{\partial f}{\partial r}\right)\left(\frac{\partial r}{\partial x}\right)$$

Fixed (same thing)

"Useful Theorem" §5.4.

$x = x(y,z)$ (not necessarily a position)

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz.$$

invert: $y = y(x,z)$

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz.$$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z \left[\left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz\right] + \left(\frac{\partial x}{\partial z}\right)_y dz.$$

$dz = 0$

$$\boxed{\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial y}{\partial x}\right)_z^{-1}}$$

$\left(\frac{\partial x}{\partial x}\right)_z = 1.$ A

$dx = 0$

$$\left(\frac{\partial x}{\partial y}\right)_z \cdot \left(\frac{\partial y}{\partial z}\right)_x + \left(\frac{\partial x}{\partial z}\right)_y = 0$$

$$\boxed{\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1}$$ B

deserves equation numbers

cyclic.

Thermodynamics: $u = u(s, v)$

$$du = \left(\frac{\partial u}{\partial s}\right)_v ds + \left(\frac{\partial u}{\partial v}\right)_s dv$$

$$T = \left(\frac{\partial u}{\partial s}\right)_v \qquad -p = \left(\frac{\partial u}{\partial v}\right)_s$$

$$\boxed{du = T\,ds - p\,dv} = \text{"}đq - đw\text{"}$$

heat flow — work done.

$đq$, $đw$ "infinitesimal", not $d(\text{something})$

Mixed 2nd derivative,

$$\frac{\partial^2 u}{\partial v\,\partial s} = \frac{\partial}{\partial v}\left(\frac{\partial u}{\partial s}\right)_v = \left(\frac{\partial T}{\partial v}\right)_s$$

$$= \frac{\partial^2 u}{\partial s\,\partial v} = \frac{\partial}{\partial s}\left(\left(\frac{\partial u}{\partial v}\right)_s\right) = -\left(\frac{\partial p}{\partial s}\right)_v$$

$$\boxed{\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v}$$

one of many "Maxwell relations"

Can turn $\underline{u}$ into a function of $\underline{T}$ and $\underline{V}$ (both measurable).

$$f = u - Ts.$$

$$df = du - T\,ds - s\,dT$$

$$= \cancel{T\,ds} - p\,dV - \cancel{T\,ds} - s\,dT$$

$$\boxed{df = -s\,dT - p\,dV}$$

$$\boxed{\left(\frac{\partial s}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V}$$

Wave equation. $\rho = \rho_0 + \Delta\rho$

$$\boxed{\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v}{\partial x} = 0} \qquad \boxed{\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}}$$

$$\frac{\partial^2 \rho}{\partial t^2} = -\rho \frac{\partial^2 v}{\partial x \partial t} = -\rho \frac{\partial^2 v}{\partial t \partial x} = + \frac{\partial^2 p}{\partial x^2}$$

$$= \frac{\partial^2}{\partial x^2}\left(p_0 + \frac{\partial p}{\partial \rho}\cdot \Delta\rho\right) = \left(\frac{\partial p}{\partial \rho}\right)\frac{\partial^2 \rho}{\partial x^2}$$

$$\boxed{\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial x^2} = 0}$$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial t}dt$$

Let $u = x + ct$ "advanced/retarded"

$v = x - ct$. "null coordinates"

$$df = \frac{\partial f}{\partial u}du + \frac{\partial f}{\partial v}dv$$

$$= \frac{\partial f}{\partial u}\left(\overset{1}{\frac{\partial u}{\partial x}}dx + \overset{c}{\frac{\partial u}{\partial t}}dt\right) + \frac{\partial f}{\partial v}\left(\overset{1}{\frac{\partial v}{\partial x}}dx + \overset{-c}{\frac{\partial v}{\partial t}}dt\right)$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}} \qquad \boxed{\frac{\partial f}{\partial t} = c\left(\frac{\partial f}{\partial u} - \frac{\partial f}{\partial v}\right)}$$

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iterate $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial x}\right) + \frac{\partial}{\partial v}\left(\frac{\partial f}{\partial x}\right)$

$$= \frac{\partial^2 f}{\partial u^2} + 2\frac{\partial^2 f}{\partial u \partial v} + \frac{\partial^2 f}{\partial v^2}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = c \cdot \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial t}\right) - c\frac{\partial}{\partial v}\left(\frac{\partial f}{\partial t}\right)$$

$$= c^2\left(\frac{\partial^2 f}{\partial u^2} - 2\frac{\partial^2 f}{\partial u \partial v} + \frac{\partial^2 f}{\partial v^2}\right)$$

wave equation $\cdot$ $\frac{\partial^2 f}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = \cancel{f_{uu}} + 2f_{uv} + \cancel{f_{vv}}$

$$- \frac{1}{c^2} \cdot c^2\left(\cancel{f_{uu}} - 2f_{uv} + \cancel{f_{vv}}\right)$$

$$= 4 f_{uv} = 4\frac{\partial^2 f}{\partial u \partial v} = 0$$

$$\frac{\partial^2 f}{\partial u \partial v} = 0 \quad \boxed{f = g(u) + h(v)}$$

$$= g(x + ct) + h(x - ct)$$