PHZ 3113 Fall 2017

Homework #3, Due Friday, September 15

1. The variables x and y are related to the variables u and v by $x = e^u \cos v$, $y = e^u \sin v$. Write the Laplacian operator $\nabla^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2}$ in the variables u and v.

2. The free energy $F(T, V)$ of an ideal gas is

$$F = -NkT \ln\Big[\frac{V}{N}\left(\frac{mkT}{2\pi\hbar}\right)^{3/2}\Big],$$

where N, k, and $\hbar$ are constants.

(a) From $dF = -S\,dT - p\,dV$, compute S and p. Do you recognize the pressure you obtain?

(b) The internal energy is given by $U = F + TS$. Compute U. Your answer might appear to be simple, but show from dU that U should be a function of S and V. Write U as a function of S and V. Compute the pressure from U.

3. The two constraints $f(x, y, z) = 0$ and $g(x, y, z) = 0$ in a three-dimensional world define a one-dimensional curve.

(a) Find dy/dx along this curve.

(b) Do you get a sensible result for the intersection of the sphere $x^2 + y^2 + z^2 = 1$ with the plane $z = 0$?

4. Find the point on the curve defined by $\frac{5}{8}\,x^2 - \frac{3}{4}\,xy + \frac{5}{8}y^2 = 1$ that is closest to the point $(x, y) = (1, -1)$.