PHZ 3113 Fall 2017

Homework #2, Due Friday, September 8

1. Write the first five terms of the series for $\cos[(1+x)^{1/3} - 1]$. Use logical reasoning or common sense to find the radius of convergence of this series.
Bonus: Find the first term for which the numerator of the expansion coefficient is not a prime number.

2. The function $i(x)$ is defined as

$$i(x) = \frac{(3+x^2)\,\sinh x - 3x\,\cosh x}{x^3}.$$

What is the behavior of $i(x)$ as $x \to 0$? As $x \to \infty$? Sketch the function $i(x)$.

3. (a) The function $f(z)$ has the series expansion

$$f = z + \frac{1}{3}\,z^3 + \frac{1}{5}\,z^5 + \frac{1}{7}\,z^7 + \cdots = \sum_{\substack{m=1 \\ \text{odd}}}^{\infty} \frac{1}{m}\,z^m.$$

Write $f(z)$ in closed form.

(b) Let z be complex, $z = x + iy$ where x and y are real. Find the imaginary part of f in its closed form in terms of x and y (the solution involves an inverse tangent).

(c) The series solution to an electrostatics problem in the interior of a cylinder of radius b is the potential

$$\Phi = \frac{4V_0}{\pi} \sum_{\substack{m=1 \\ \text{odd}}}^{\infty} \frac{1}{m}\,(\frac{\rho}{b})^m\,\sin m\phi,$$

where ρ and ϕ are polar coordinates. Show how this is related to (a), and use (b) to write Φ in closed form. What is the potential on the surface $\rho \to b$?

4. A thin tube of length ℓ and cross-sectional area A, sealed on both ends, is divided into two halves by a small drop of mercury of mass m. The two parts of the tube contain air at constant temperature T_0. In equilibrium, when the air on both sides is at the same pressure p_0, the drop of mercury is located a distance x_0 from the left end of the tube. When the tube is disturbed, the drop of mercury executes small oscillations about its equilibrium position. What is the period of these oscillations?