PHZ 3113 Fall 2017

Homework #1, Due Friday, September 1

1. Let $a_n = \left(1 + \frac{\alpha}{n}\right)^n$. What is $\lim_{n\to 0} a_n$? What is $\lim_{n\to\infty} a_n$?

2. Decide whether the following sums converge or diverge. (Provide reasons for your choices.)

$$(a) \quad \sum_{n=2}^{\infty} \frac{1}{n\,\ln n \ln(\ln n)} \qquad (b) \quad \sum_{n=1}^{\infty} \frac{\sin n}{n} \qquad (c) \quad \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$$

3. Consider the following alternating series

$$\frac{2}{1} - \frac{1}{2} + \frac{2}{3} - \frac{1}{4} + \frac{2}{5} - \frac{1}{6} + \frac{2}{7} - \frac{1}{8} + \frac{2}{9} - \frac{1}{10} + \cdots.$$

Does the general term go to zero? Does the "Test for alternating series" in Section 4.3.3 apply? Does the series converge?

4. For what values of p does the sum $\zeta(p) = \sum_{n=1}^{\infty} \frac{1}{n^p}$ converge? In terms of $\zeta(p)$, what is the sum over only even integers? Only odd integers? What is $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}$?

5. The *partition function* for a quantum harmonic oscillator with energy levels $\epsilon_n = (n+\frac{1}{2})\,\hbar\omega$ is $Z = \sum_{n=0}^{\infty} e^{-\beta\epsilon_n} = \sum_{n=0}^{\infty} e^{-(n+\frac{1}{2})\beta\hbar\omega}$. Compute Z.

6. Take the alternating harmonic series $\sum(-1)^{n+1}/n$ and sum it in the following order: take the first two two (positive) odd terms and the first (negative) even term; then the next two odd terms and the next even term, two more odd terms and one even term, and so on. Does the sum in this ordering converge? If so, what is the resulting value of the sum? Since infinite precision, even if achievable, is not practical, make some choices and include an estimate of the accuracy of your result.