9/12/18 NOTES

GRANT ELLIOT AND LUIS ORTEGA

1. Tensors

A tensor $\mathbb{T} = T^{\mu\nu}_{\sigma}\vec{e}_\mu\vec{e}_\nu\vec{e}^\sigma = T^{\mu\nu}_{\sigma}\partial_\mu\partial_\nu dx^\sigma = T^{\alpha\beta}_{\gamma}\partial_\alpha\partial_\beta dx^\gamma$ is usually referred to by its components which are determined by the tensors action on vectors as shown below.

$$T^{\mu\nu}_{\sigma} = \mathbb{T}(dx^\mu, dx^\nu, \partial_\sigma) = T^{\alpha\beta}_{\gamma}\delta^\mu_\alpha\delta^\nu_\beta\delta^\gamma_\sigma = T^{\mu\nu}_{\sigma}$$

Under a coordinate transformation $x^\mu \to x^{\mu'} = x^\mu + \epsilon\xi^\mu$ the metric at some point can be expanded as around a point O as

$$g_{\mu\nu}|_o - \epsilon\frac{\partial g_{\mu\nu}}{\partial x^\sigma} + O(\epsilon^2)$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = ds^2|_o + \epsilon(...)\xi^\mu$$

2. Causality

Given a spacelike hypersurface Σ and data on a subset S of Σ, only the future lightcone of S can be predicted. If the surface S has a hole in it, then the the future of the missing portion must be excluded from the future cone of S. Given data on S a point Q in the future can only be determined if every past timelike curve intersects S

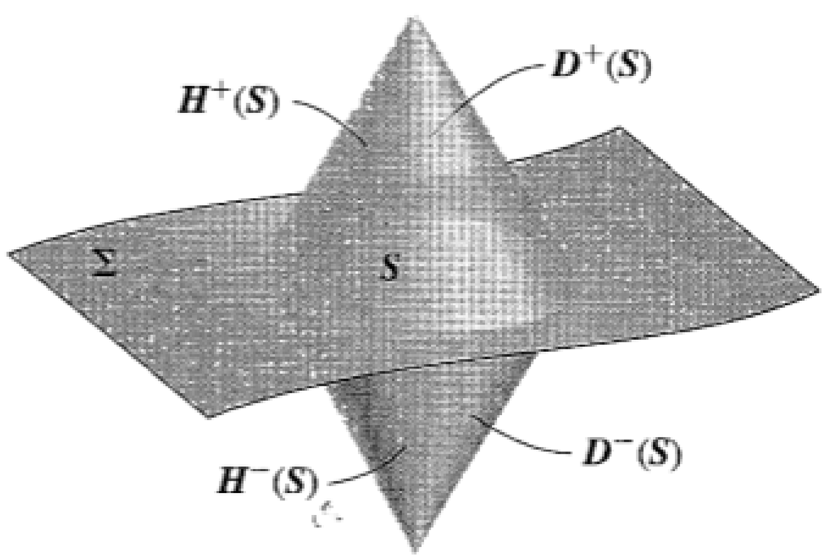

Figure 1

A casual curve is either timelike or null. If all casual curves from p intersects S, then p is in $D^+(S)$. $D^+(S)$ is called the future domain of dependence of S The boundary of $D^+(S)$ is $H^+(S)$ Note that as S gets larger, $D^+(S)$ gets larger. Does $D^+(\Sigma)$ exists? It turns out that some infinite spacetimes cannot have their futures predicted. This is not the case for Minkowski space, its entire future can be predicted. That is $D^+(\Sigma) \cup D^-(\Sigma) = \Sigma$

Some points can have futures that do not intersect, that is they are always spacelike seperated. This is shown in Figure 2. This can happen with the metric

$$ds^2 = -c^2dt^2 + t^{2q}(dx^2 + dy^2 + dz^2)$$

which has null geodesics governed by $-c^2dt^2 + t^q dx^2 = 0$

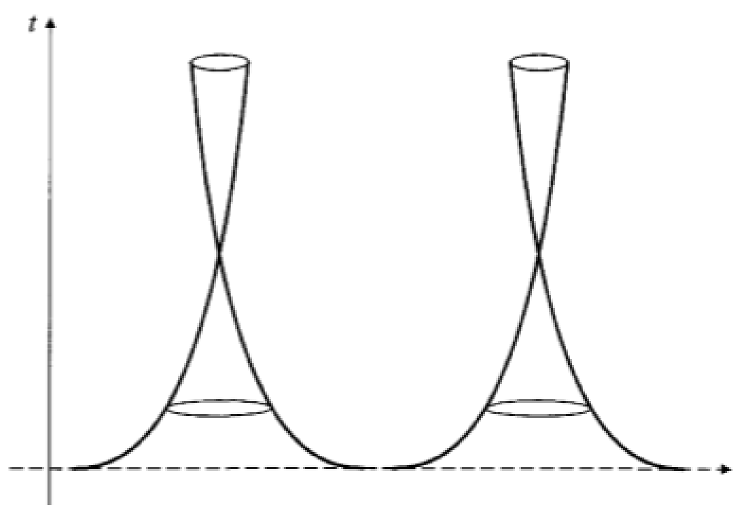

FIGURE 2

A space that is completely determined by a spacelike hypersurface is called globally hyperbolic, an example of which is Minkowski space. De Sitter space is not globally hyperbolic. Minkowski space can be compactly drawn as shown in Figure 3. The spatial and time range are drawn in at a finite length.

Light cones can become tipped over, one example of this is a cylindrical spacetime which has closed timelike curves as shown in Figure 4.

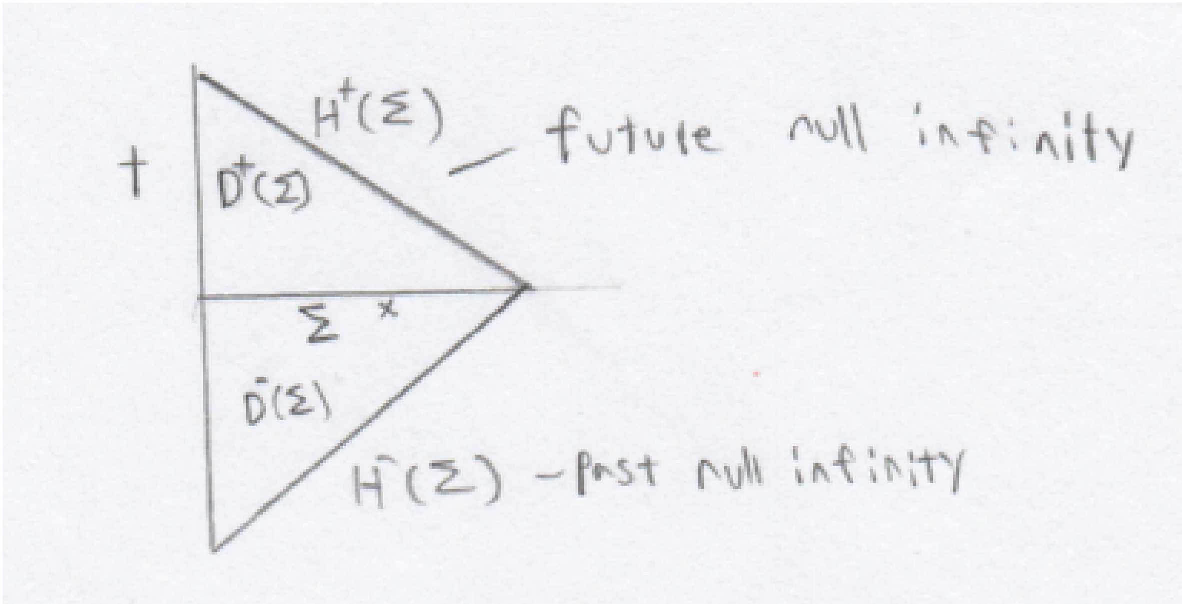

Figure 3

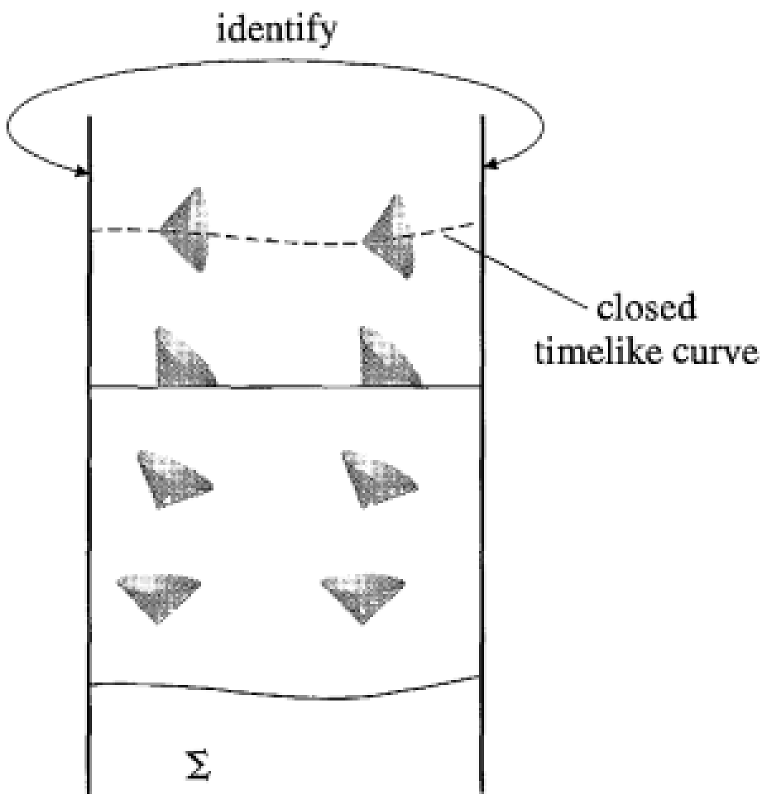

Figure 4

Black holes can cause gradually tilting of light cones, once beyond the event horizon, all timelike curves are inside the event horizon. Light cannot diverge to infinity inside the horizon. Lightcones far away from the black hole are unaffected. The spacetime of a collapsing body is globally hyperbolic. Unlike Minkowski space, collapsing body has an the horizon as a boundary, as shown in figure 6. Rotating black holes can cause closed timelike curves as well.

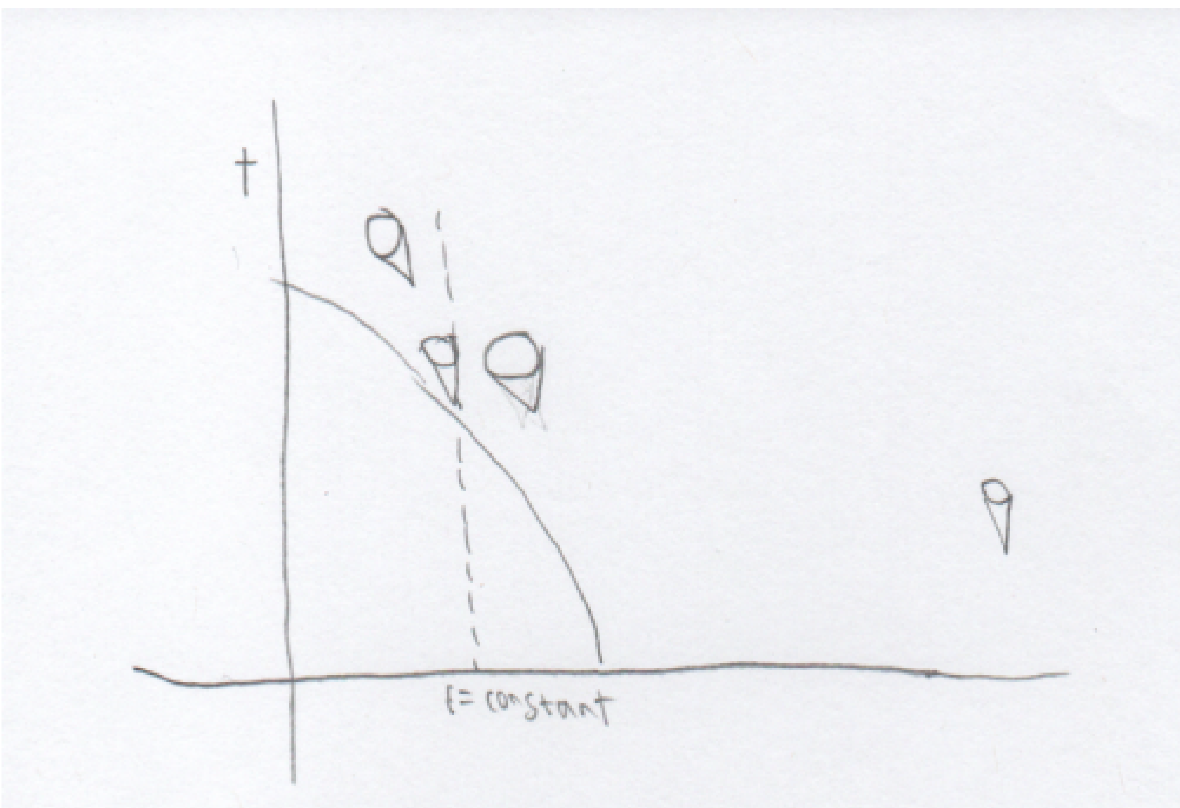

FIGURE 5

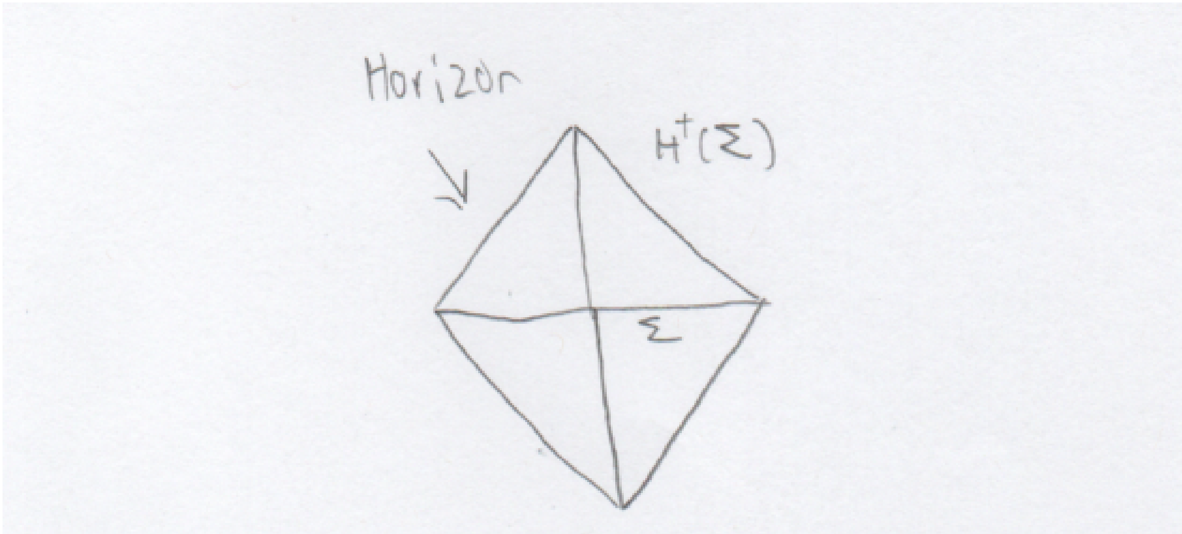

FIGURE 6