Special and General Relativity I
Fall 2018

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1. Wednesday, August 22
Introduction of Differential Geometry

Consider a scalar field, temperature T($\vec{x}$) or distance D($\vec{x}$). Compare with vectors $\vec{V}(t)$, $\vec{A}(t)$ which can be expressed differently in different coordinate systems.

Things like the electric or magnetic fields are vector fields, $\vec{E}(\vec{x},t)$ and $\vec{B}(\vec{x},t)$. They differ from velocity and acceleration in that $\vec{V}(t)$ and $\vec{A}(t)$ only have support in the worldline whereas $\vec{E}(\vec{x},t)$ and $\vec{B}(\vec{x},t)$ have support everywhere. In order to understand the behavior of vector fields in changing coordinate systems, we introduce the vector potential

$A_\mu(\vec{x},t)$ (μ= 0,1,2,3) and $A_0 = \Phi$

We must also consider the constraints from Maxwell's equations to these vector fields, i.e.

$\nabla \cdot B = 0$
$\nabla \cdot E = \rho$

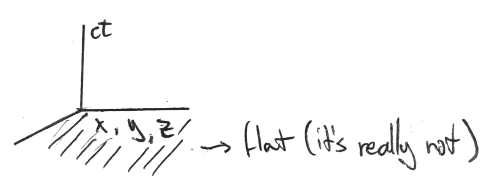
In general, Hamiltonian system's constraints are of the form

$F(\phi^\mu, \pi_\mu) = 0$ $\dot{\phi}^\mu = F(\_\_\_\_)$ $\dot{\pi}_\mu = F(\_\_\_\_)$

For the vector potential $A_\mu(\vec{x},t)$, the constraints result in two independent components. The equations of motion do not determine the time derivative of A_0, so there is gauge freedom.

This is the 4-dimensional invariance of electromagnetic fields which makes itself evident when expressing A_μ, Φ.

Special relativity states that the spatial coordinate x, y, z and time coordinate ct are part of a single spacetime. Our experience of spacetime is largely described by projection to the spatial components; the electric and magnetic fields are experienced different in frames moving with respect to one another. In special relativity we explore this quality,



$x^\mu = (ct, x, y, z)$

This quality of spacetime is a requirement for describing the experience of particles in these fields. It is important to note the the vector potential does not have an invariant meaning.

Einstein spent some time accommodating for curved spacetime (gravity), resulting in general relativity.