

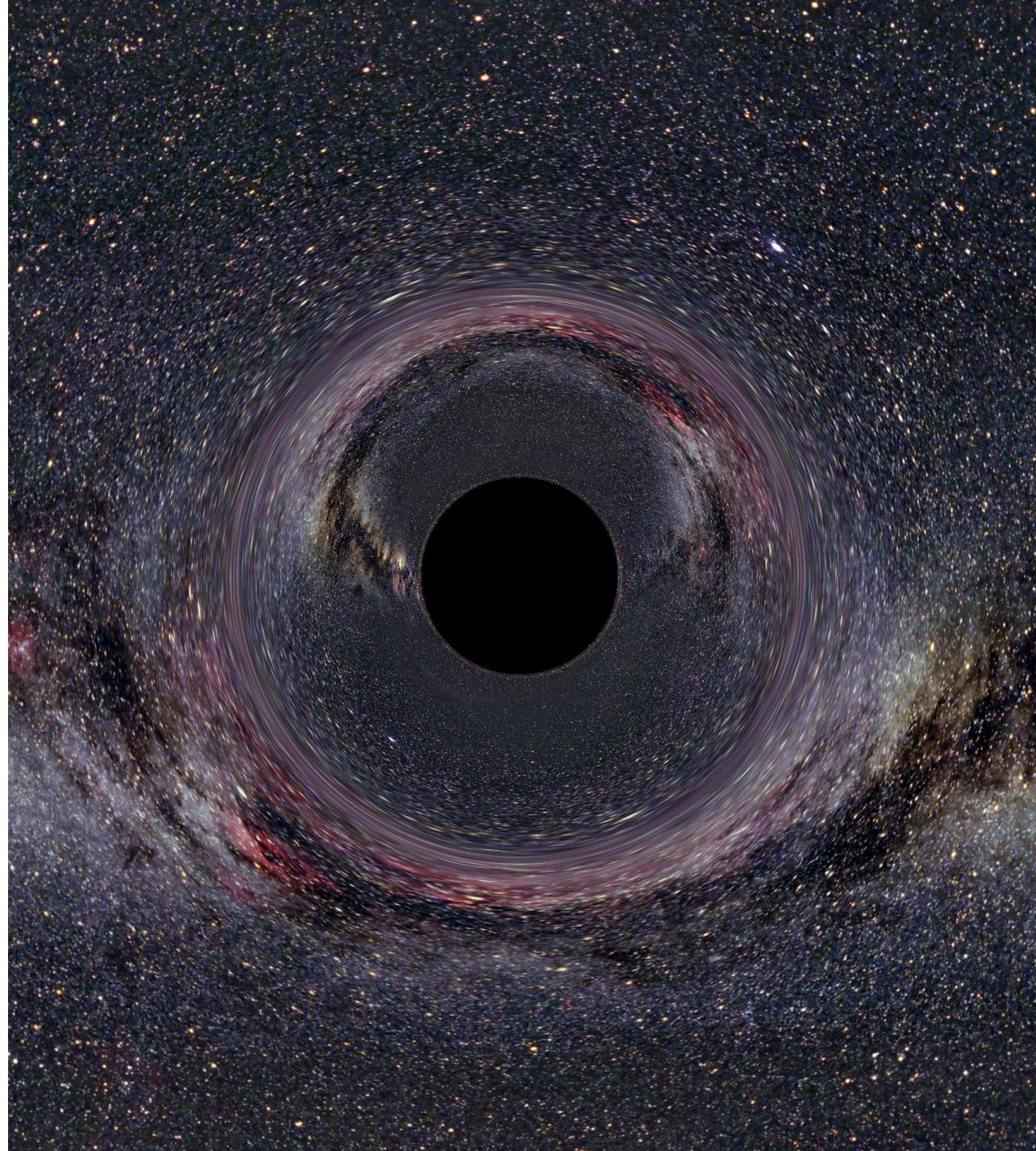
Lecture XII.

Black holes

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No-hair theorem

A stable BH only has 3 independent physical properties:

1. mass
2. angular momentum
3. charge.

These properties are observable from outside.

→ Loss of information upon infall.

Only mass → Schwarzschild BH

Mass + angular momentum → Kerr BH

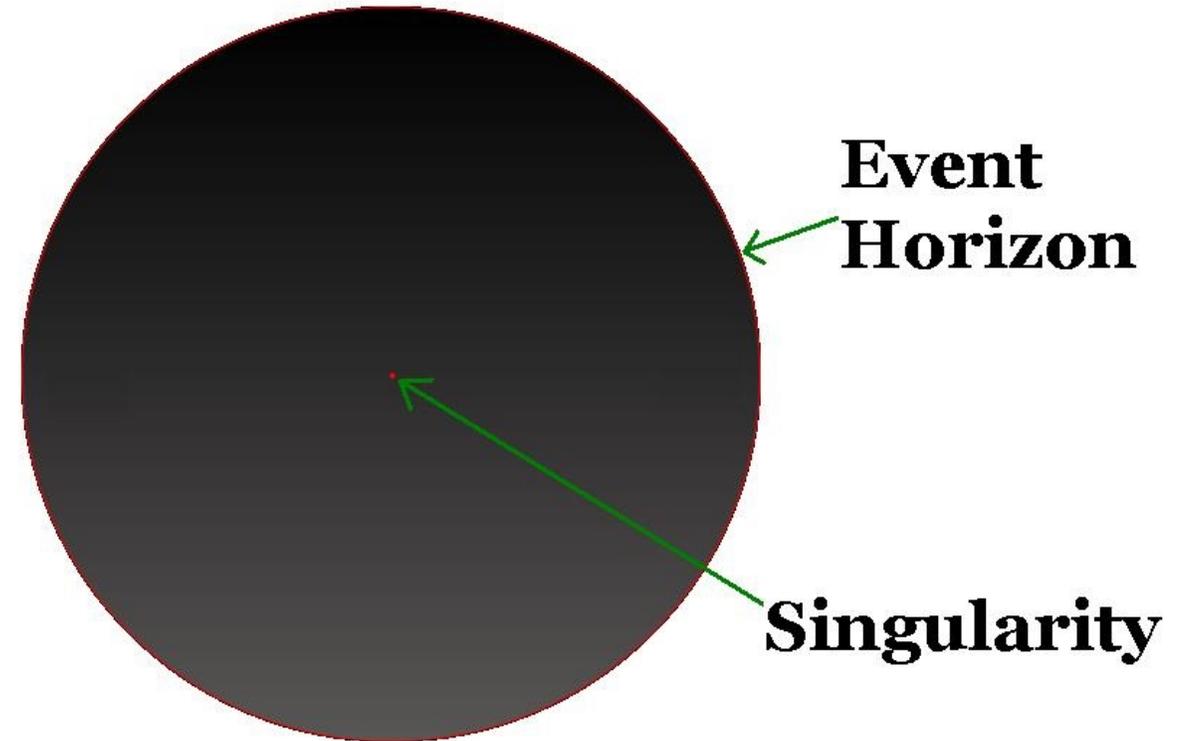
Mass + charge → Reissner-Norström BH

Mass + angular mom. + charge → Kerr-Newman BH

Far from the horizon, the BH grav. field is identical to that of any other object with the same mass.

Limit on angular momentum and charge:

$$Q^2 + \left(\frac{J}{M}\right)^2 \leq M^2$$



Schwarzschild radius

$$R = \frac{2GM}{c^2}$$

(Wikipedia)

Formation mechanisms?

Micro-black holes

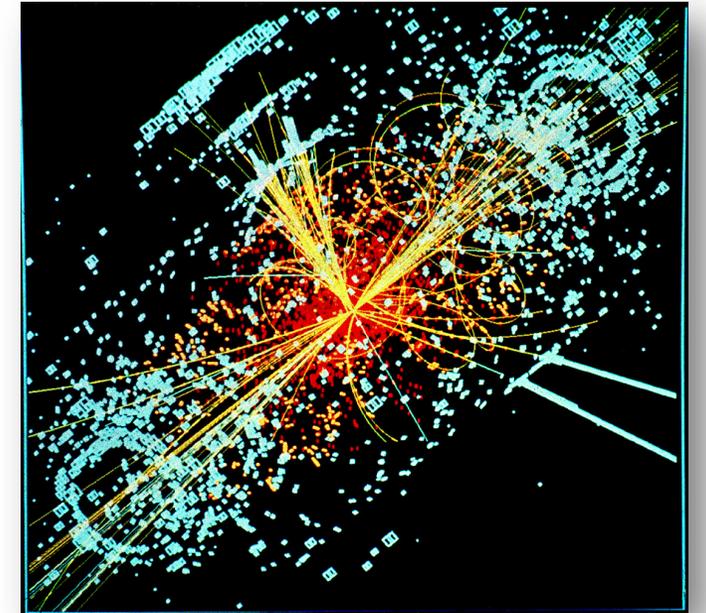
Schwarzschild radius: $R = 2GM/c^2$

Compton wavelength: $\lambda = h/Mc$

$M \geq \text{Planck mass}$

In some extensions of present physics (e.g. string theory) gravity can increase faster at short distances \rightarrow lower minimum BH mass

 The Large Hadron Collider (LHC) and cosmic rays could produce BHs!



If such a small BH was created on Earth, would it be dangerous?

Hawking radiation

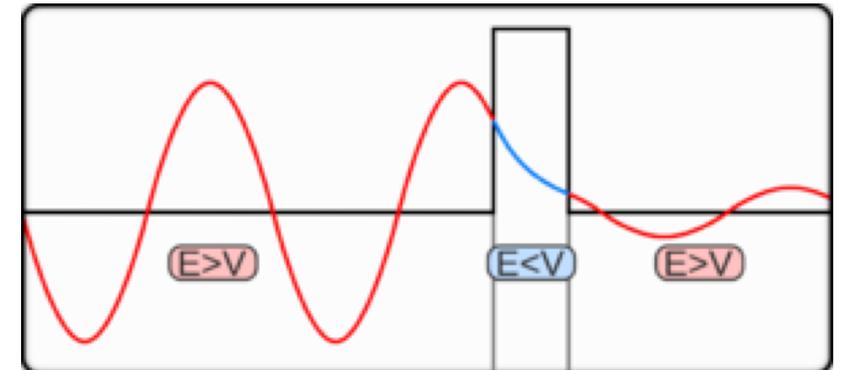
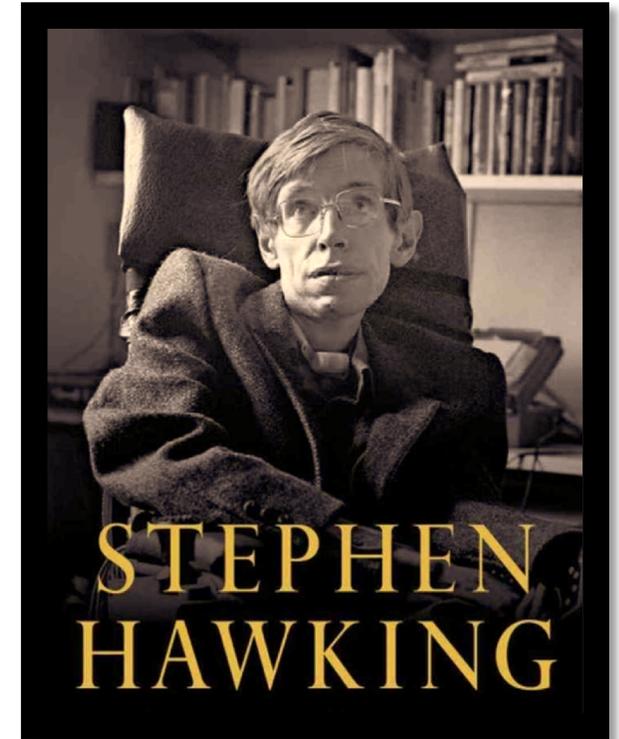
Black hole “temperature”:

$$T = \frac{\hbar c^3}{8\pi GMk_B} \left(\approx \frac{1.227 \times 10^{23} \text{ kg}}{M} \text{ K} = 6.169 \times 10^{-8} \text{ K} \times \frac{M_\odot}{M} \right)$$

Black hole will emit black body radiation at this temperature.

Irrelevant for astrophysical BHs.

Relevant for BH masses below 10^{12} kg.



Hawking radiation

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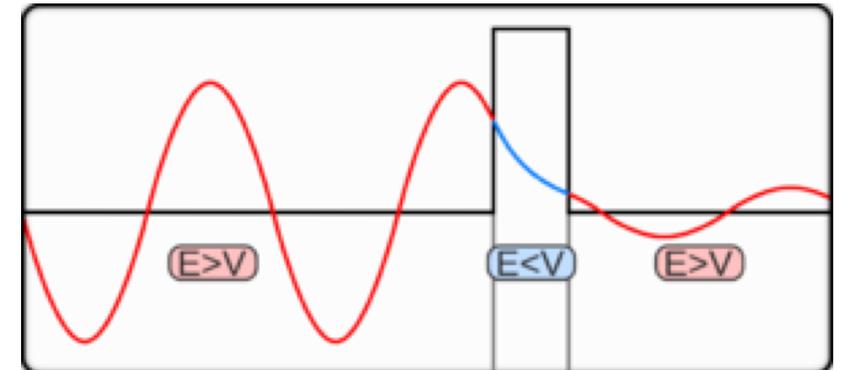
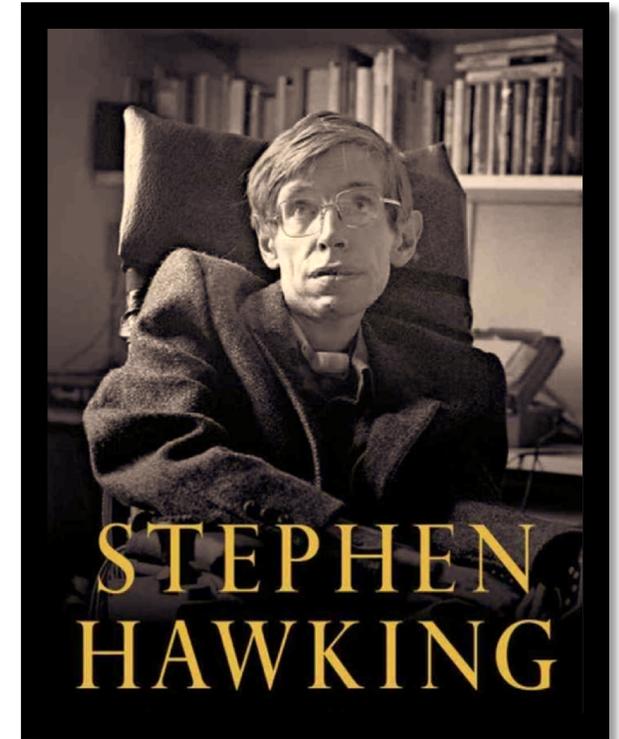
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HW: What is the heaviest BH that, created on Earth, you would survive?

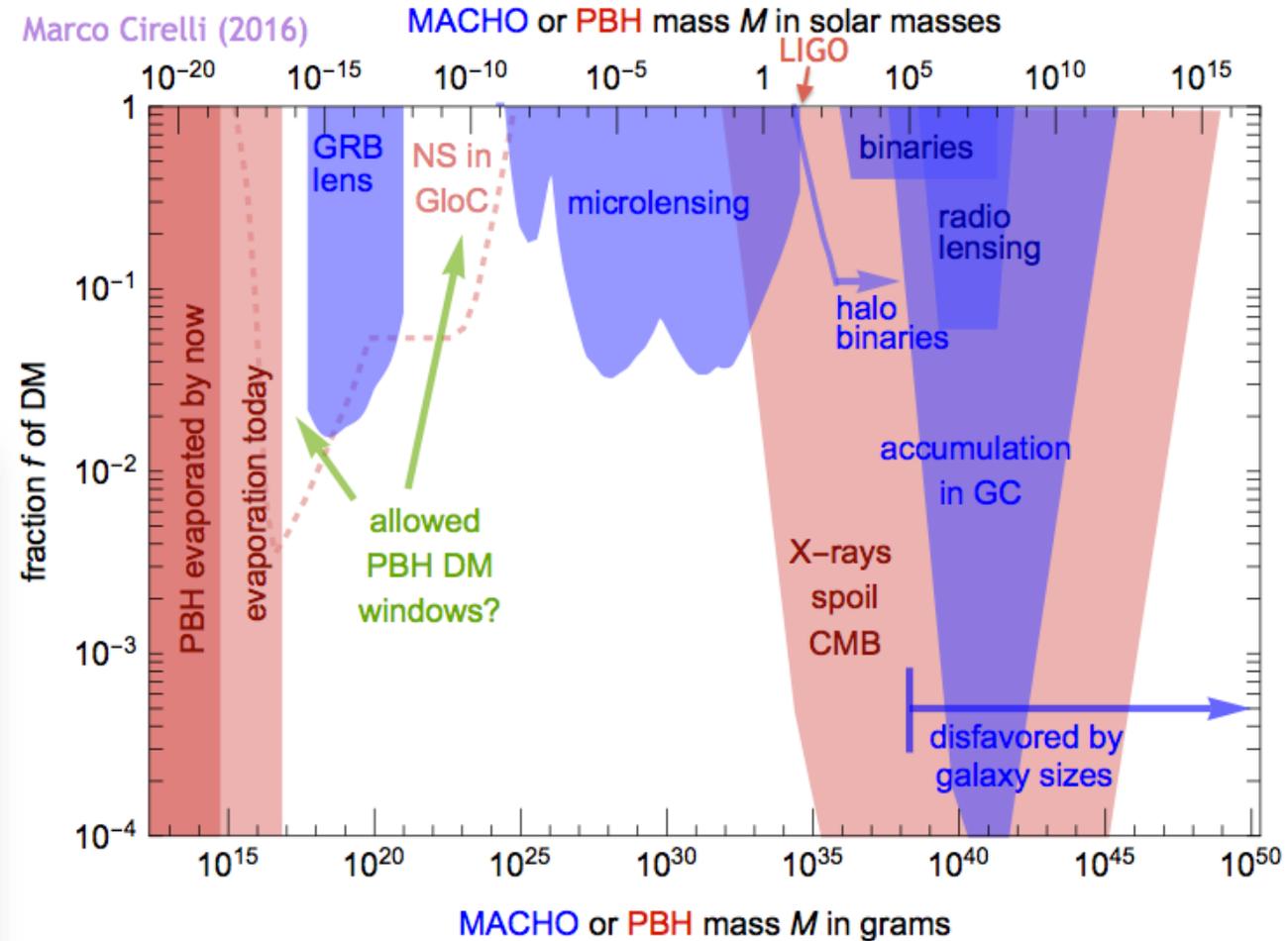
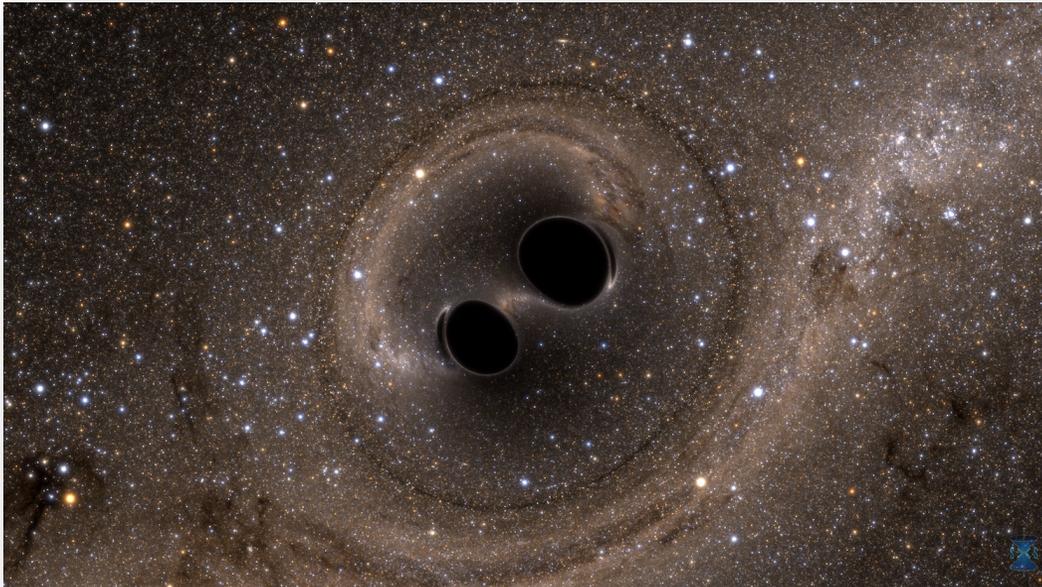


Primordial black holes

Large densities and density fluctuations soon after the big bang.

Mass distribution should be different from that of astrophysical black holes.

Primordial BHs were suggested as Dark Matter, and the origin of some LIGO BBH mergers, e.g. GW150914 (Bird+ 2016)



Black hole mass distribution

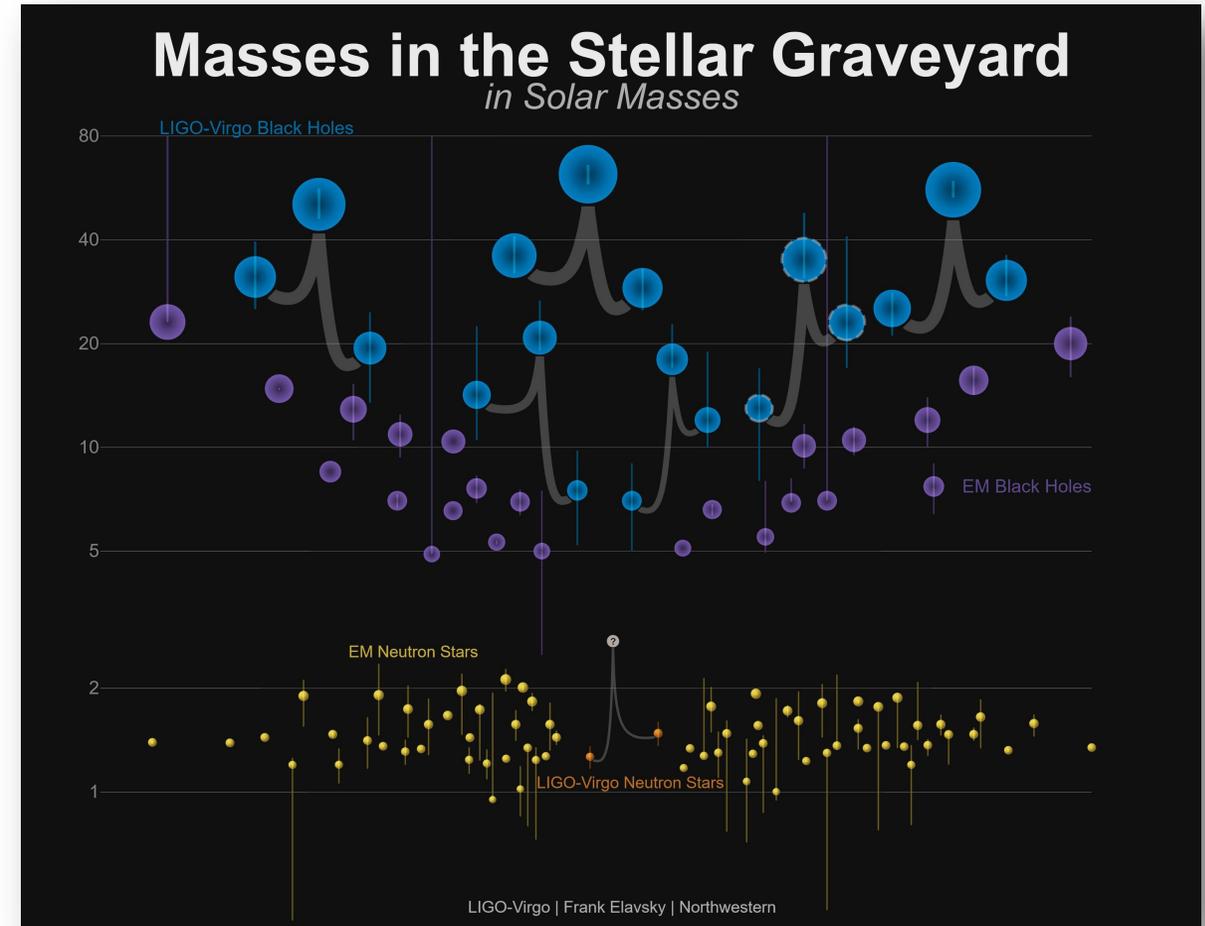
Three classes with different origin / evolution

- Stellar-mass (3 Msun – ~100 Msun)
- Intermediate-mass (~100 Msun – 10^5 Msun)
- Supermassive (10^5 Msun – 10^{11} Msun)

Stellar mass mass distribution:

- We don't know
- There seems to be a mass lower limit at 5 Msun
→ mass gap
- Best guess – Salpeter function (PDF $\sim M^{-2.3}$)

LIGO's detected BH mass distribution is consistent with Salpeter function up to a cutoff mass of ~ 50 Msun



Intermediate mass black holes

No confirmed observation.

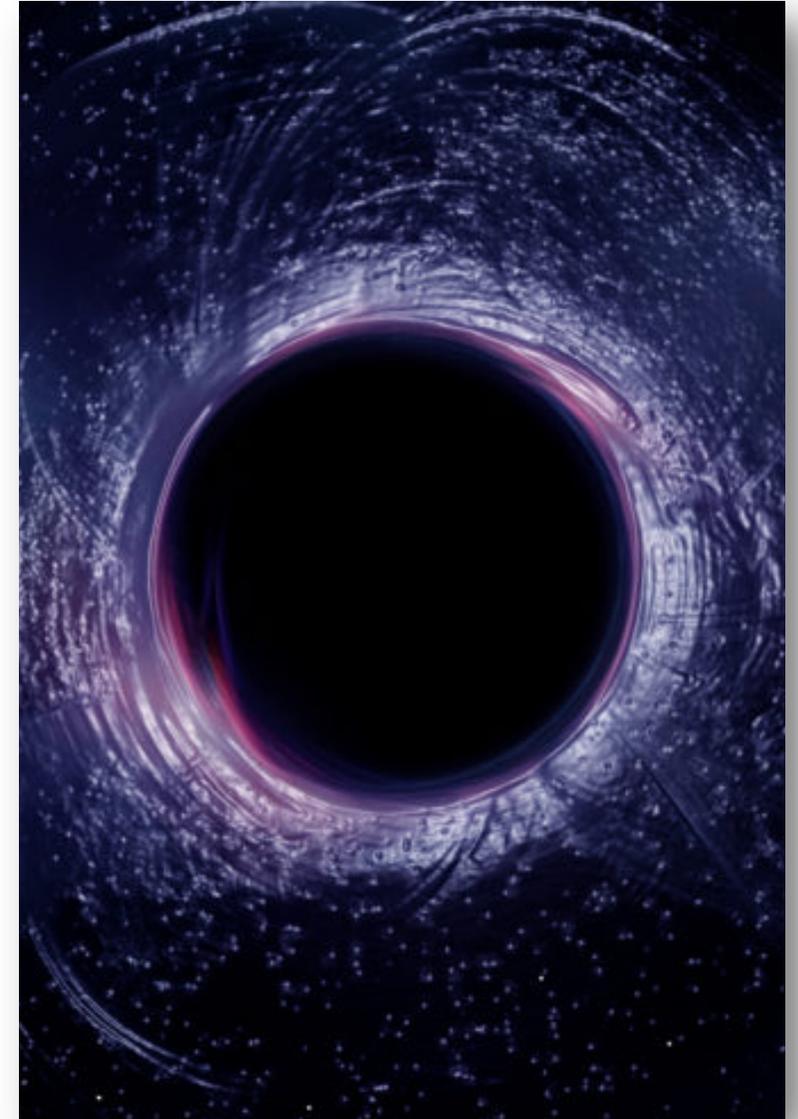
Claims:

- Measurement of Doppler shift of stellar radiation in X-ray binaries.
- Super-Eddington radiation in X-ray binaries.
- Stellar dynamics in globular clusters.
- ...

LIGO has limits on their abundance.

Origin:

- From stellar mass BHs through accretion
- Collision of multiple stars or stellar remnants in dense environments
- Primordial BHs
- Collapse of Pop III stars



Supermassive black holes

Formation: needs a seed

- Very massive star collapses
- Primordial black hole

But we don't know.

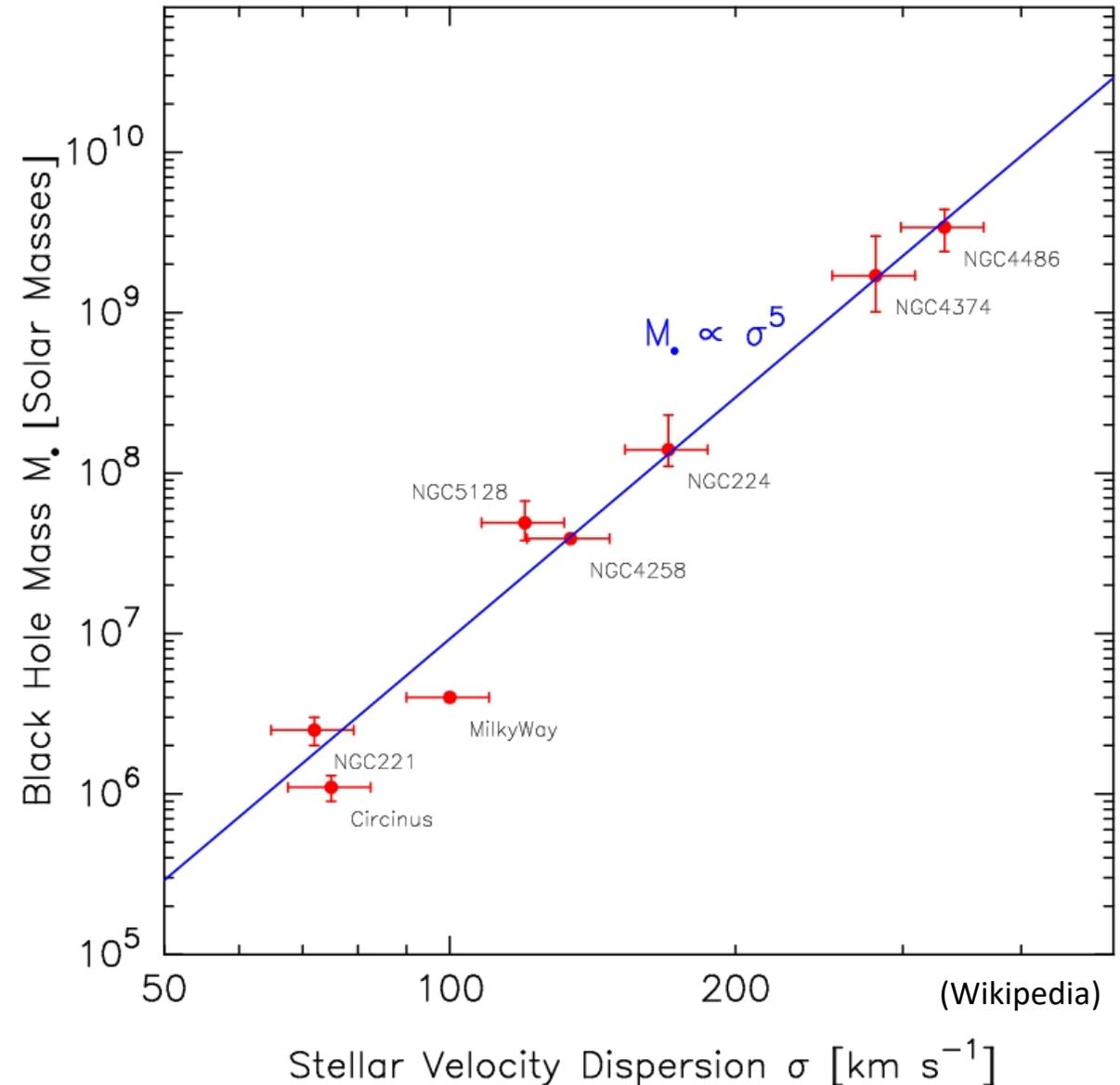
Growth:

- Accretion
- Merger with other black holes

But we don't really know.

They are fundamental components of galaxies:

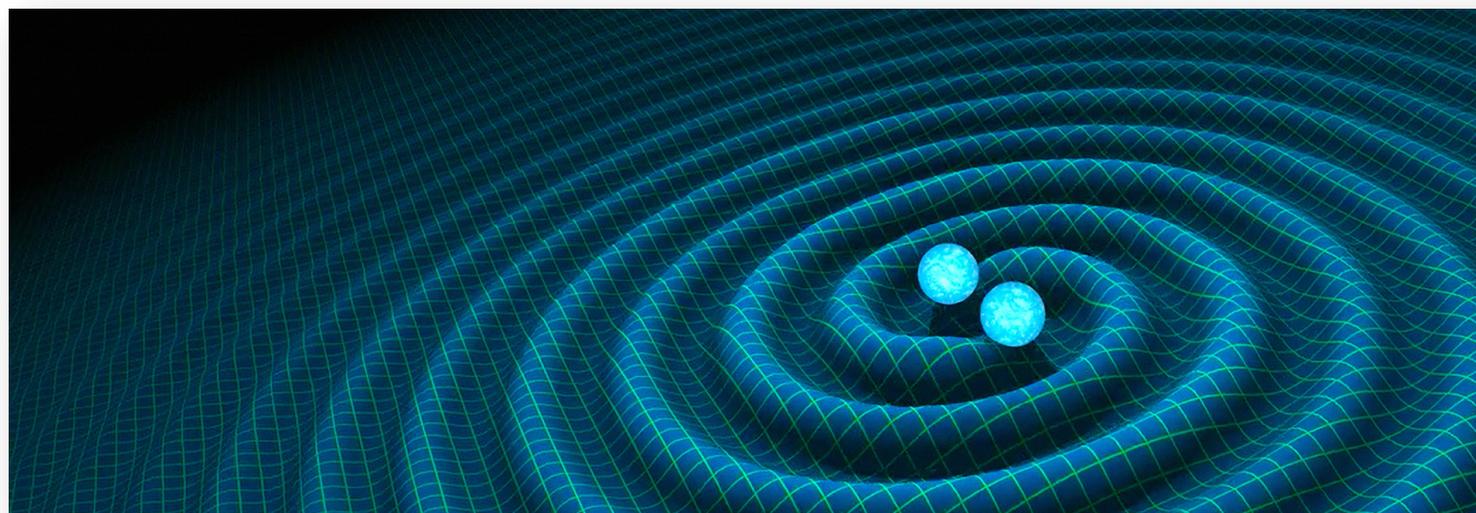
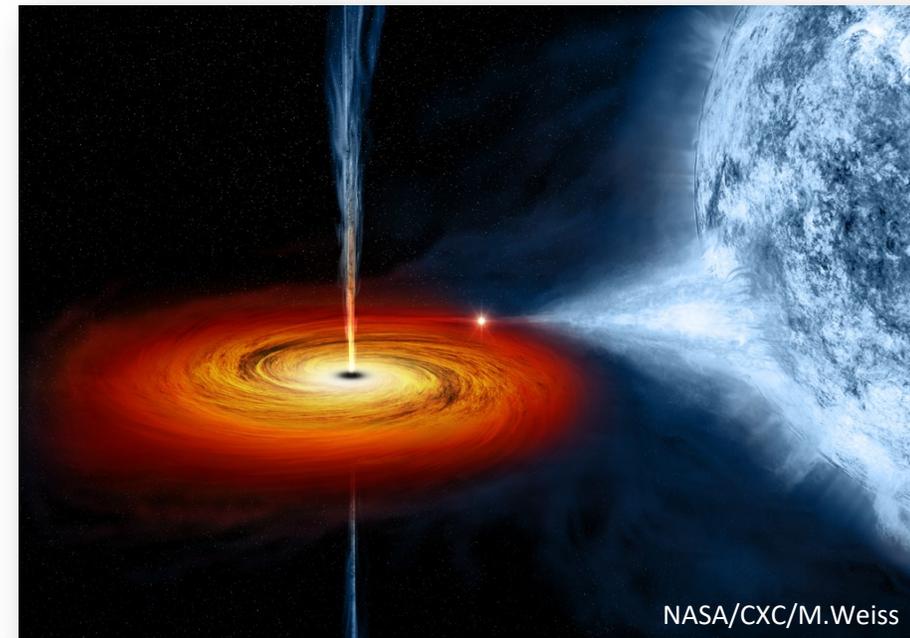
- M-sigma relation.



Observations

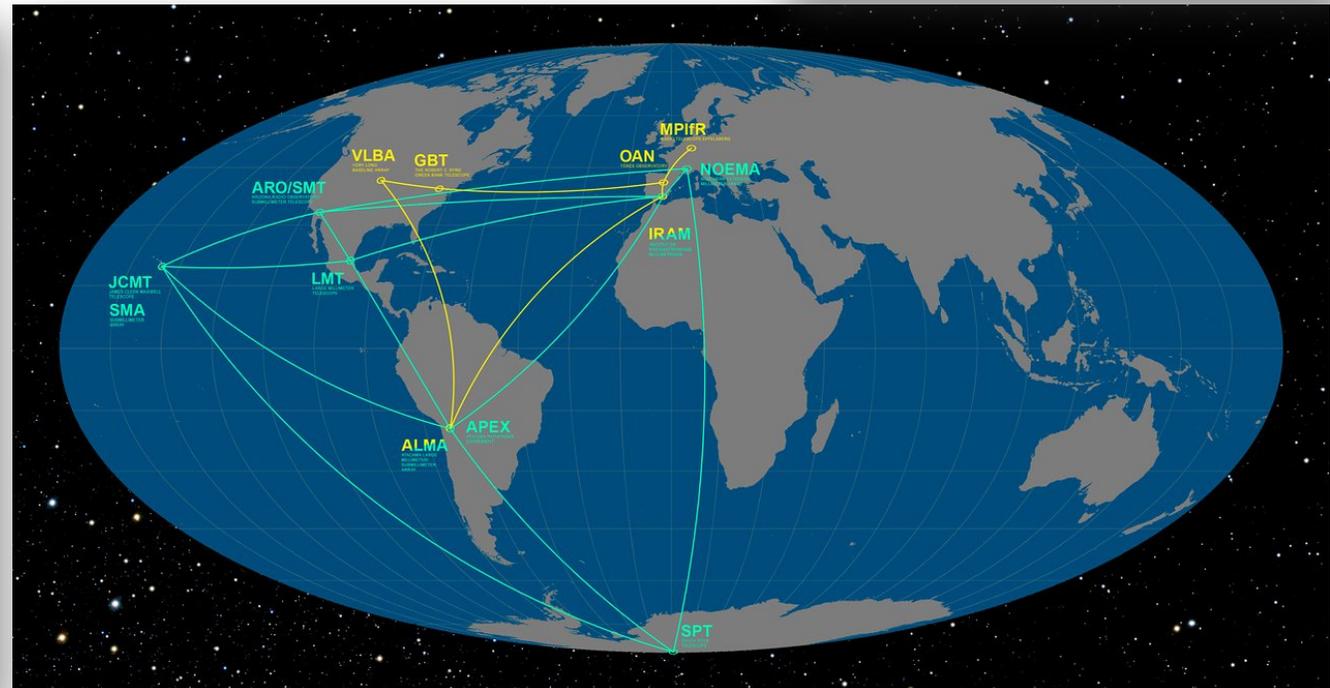
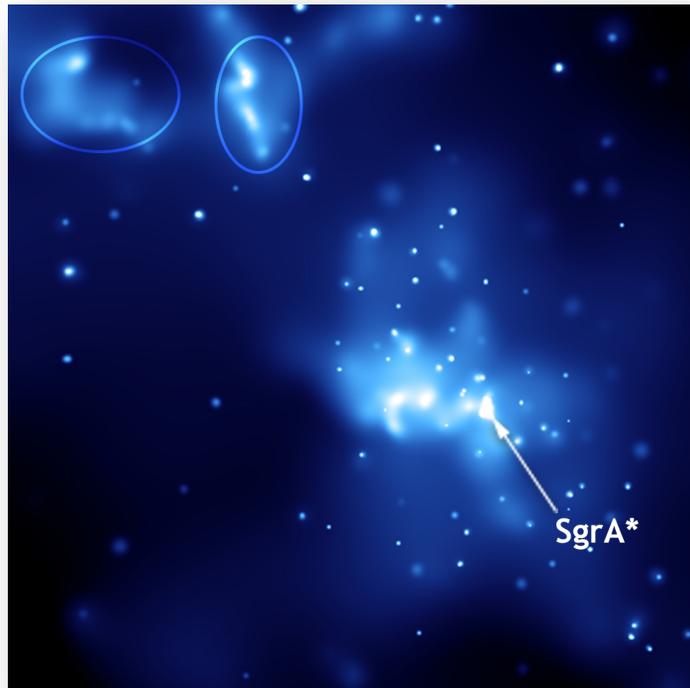
Hawking radiation is undetectable.

- Gravitational waves!
- Motion of stars orbiting Sagittarius A*
- Accretion of matter
- Gravitational lensing (not yet observed)



Event Horizon Telescope

- Network of radio telescopes.
- Interferometry
- Observe the immediate environment of the SMBH in the Milky Way (Sagittarius A*); and maybe others.
- Can resolve distance on the order of the Schwarzschild radius.
- Results expected soon...



Collisionless spherical accretion

- Collisionless gas
- Uniform dist. (spherically symmetric inflow)
- Approximation.

Loss cone – particles that get within the loss cone are accreted.



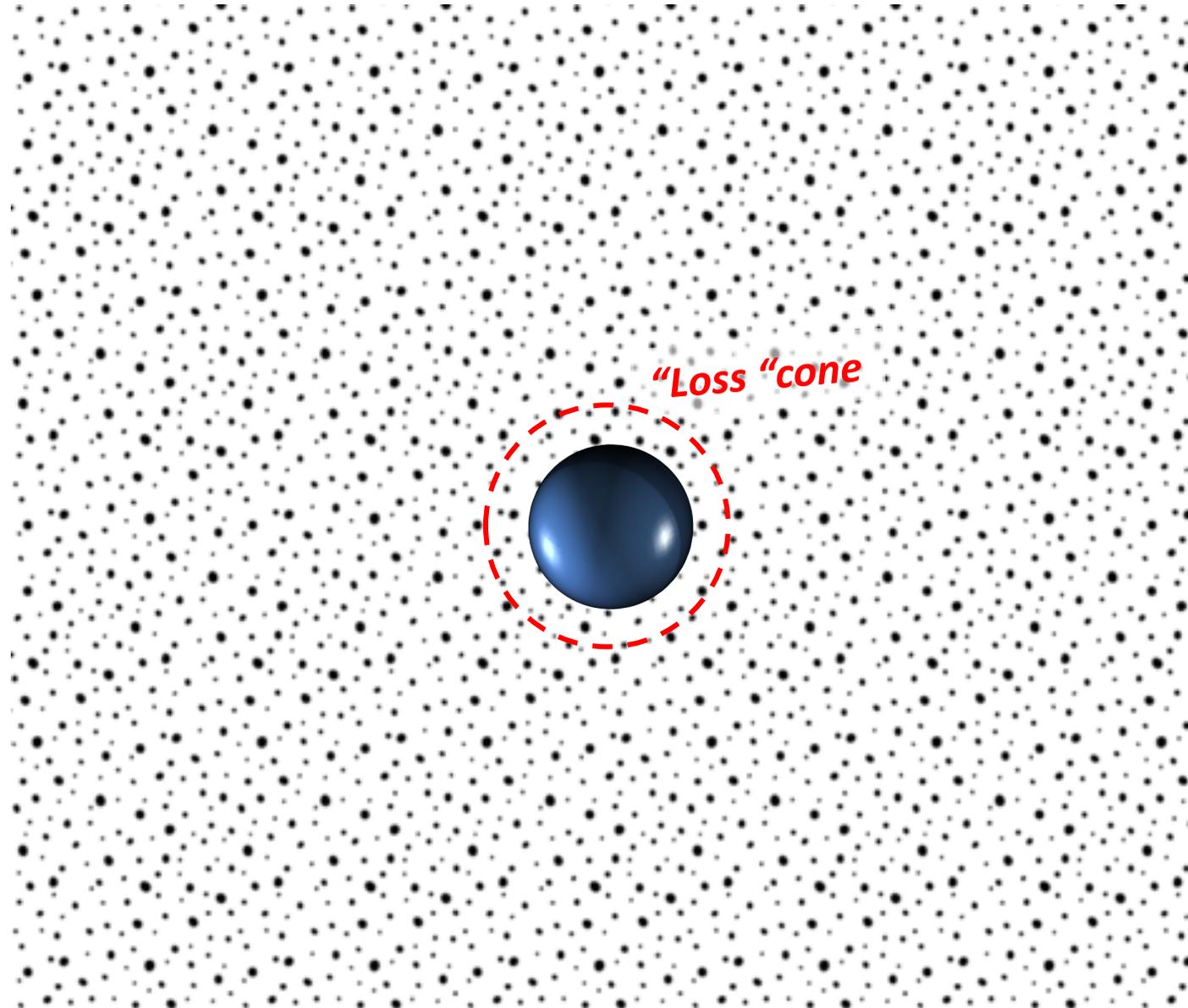
Defined w.r.t. min. angular momentum! (not radius)

$$J_{\min}(E) \equiv \left[2 \left(E + \frac{GM}{R} \right) \right]^{1/2} R \quad (\text{NR particles; Newtonian star})$$

$$J_{\min}(E) = \frac{4GM}{c} \quad (\text{NR particles; black hole})$$

$$\dot{M}(E > 0) = m\dot{N}(E > 0)$$

$$= \begin{cases} 2\pi GM^2 \rho_{\infty} v_{\infty}^{-1} \frac{R}{M} \left(1 + \frac{v_{\infty}^2 R}{2MG} \right) \\ \quad (\text{NR particles; Newtonian star}), \\ 16\pi (GM)^2 \rho_{\infty} v_{\infty}^{-1} c^{-2} \\ \quad (\text{NR particles; black hole}), \end{cases}$$



Hydrodynamic spherical accretion onto a black hole

- Interactions between gas molecules are important
- Collisions and/or macroscopically weak magnetic fields

$$\dot{M} = 4\pi\lambda_s (GM)^2 \rho_\infty a_\infty^{-1} c^{-2} \frac{c^2}{a_\infty^2}$$

O(1) constant (pointing to λ_s)
sound speed (pointing to a_∞^2)

(c.f. collisionless gas $16\pi(GM)^2 \rho_\infty v_\infty^{-1} c^{-2}$)

$\sim 10^9$ higher than in the collisionless case!

Hydrodynamic spherical accretion onto a black hole

Bondi radius --- defines **effective area** of accretion, where the black hole's escape velocity is the same as the thermal velocity of the gas.:

$$r_B = \frac{2GM}{c_\infty^2}$$

Bondi accretion:

$$\dot{M}_{\text{BH}} = \frac{4\pi G^2 M^2 \rho_\infty}{c_\infty^3}$$

Bondi-Hoyle-Lyttleton accretion:

$$\dot{M}_{\text{BH}} = \frac{4\pi G^2 M^2 \rho_\infty}{(c_\infty^2 + v_\infty^2)^{3/2}}$$

Where does spherical accretion occur?

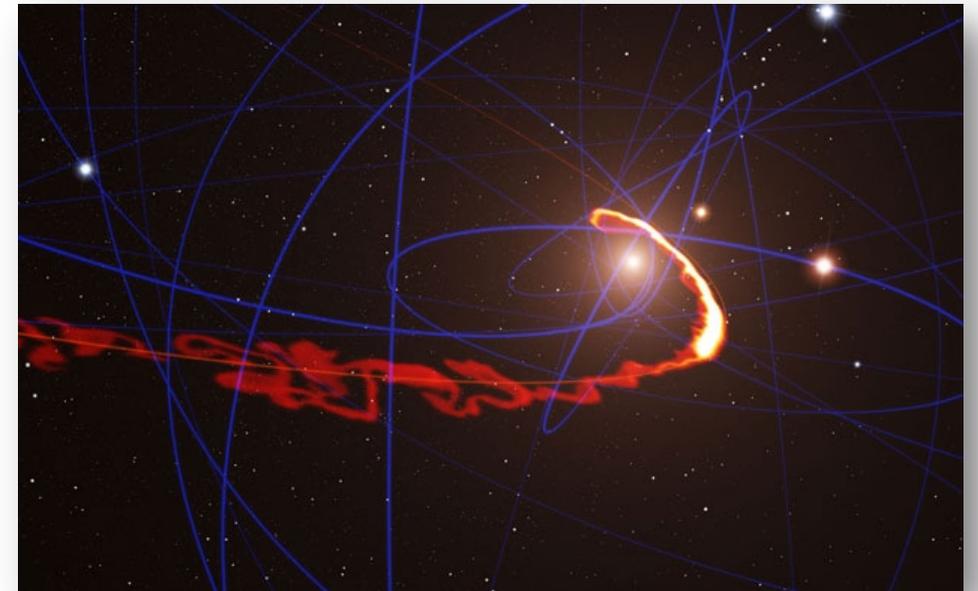
It is rare --- typically there is angular momentum.

- Planet formation from protoplanetary disk
- Accretion of interstellar gas onto a solar-mass BH

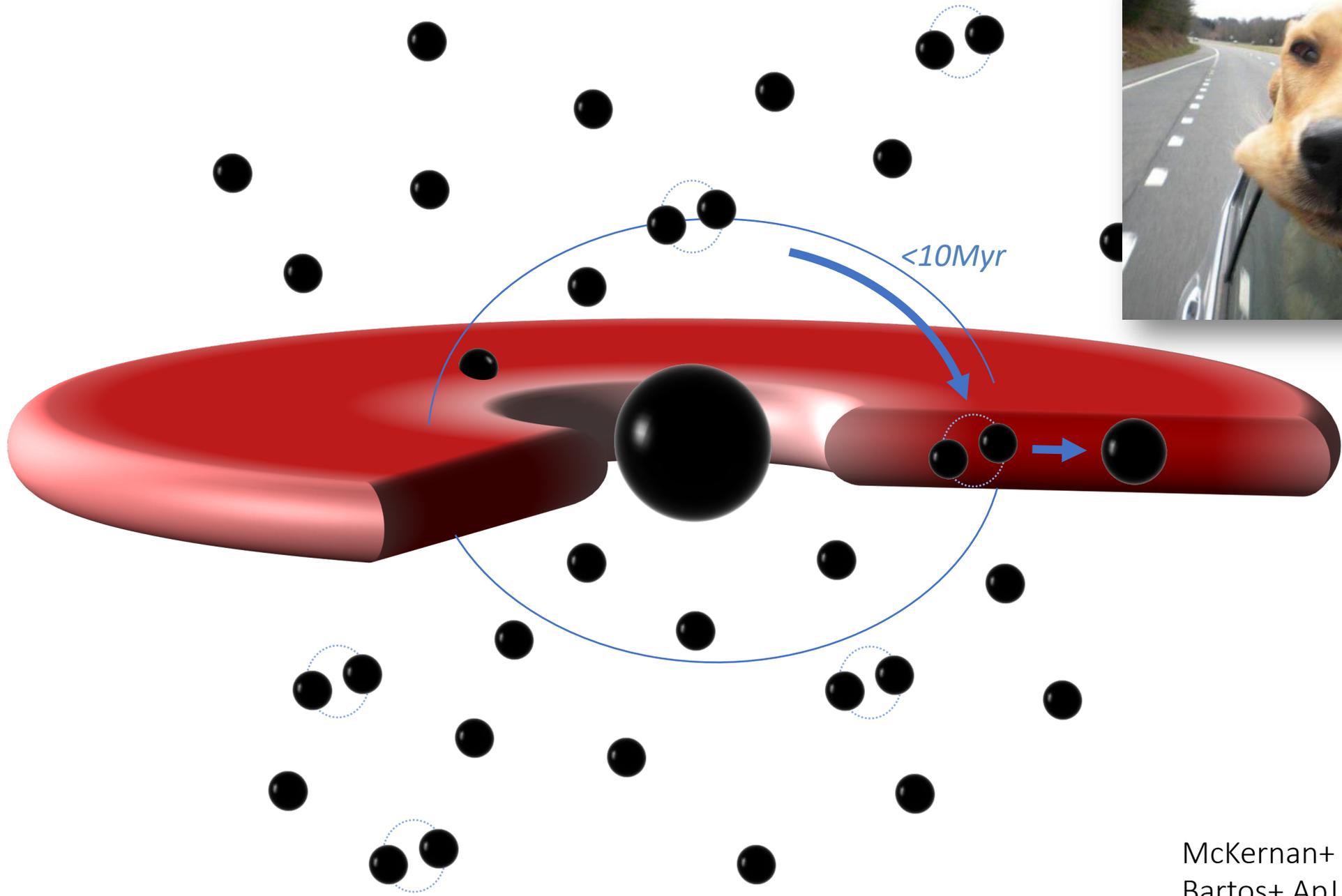
$$\dot{M} = 8.77 \times 10^{-16} \left(\frac{M}{M_{\odot}} \right)^2 \left(\frac{\rho_{\infty}}{10^{-24} \text{ g cm}^{-3}} \right) \left(\frac{a_{\infty}}{10 \text{ km s}^{-1}} \right)^{-3} M_{\odot} \text{ yr}^{-1}$$

Shapiro & Teukolsky (14.3)

- Black hole + gas cloud encounter



Nature



McKernan+ MNRAS 2012
Bartos+ ApJ 2017

Radiation from spherical accretion

- Gas heats up as it falls in \rightarrow radiates (mainly thermal Bremsstrahlung radiation)
- Most emission will come from the innermost part of the accretion where the gas is hottest.

$$L_{\text{ff}} = 1.2 \times 10^{21} \left(\frac{n_{\infty}}{1 \text{ cm}^{-3}} \right)^2 \left(\frac{T_{\infty}}{10^4 \text{ K}} \right)^{-3} \left(\frac{M}{M_{\odot}} \right)^3 \text{ erg s}^{-1}$$

- Mostly hard X-rays and gamma rays (up to ~ 10 MeV) for 1 solar mass BHs.
- Bigger BH \rightarrow **softer** spectrum
- Very low efficiency for BHs! (Spherical accretion on to NSs can have 10% efficiency!)

$$\varepsilon = \frac{L_{\text{ff}}}{\dot{M}c^2} \sim 6 \times 10^{-11} \left(\frac{n_{\infty}}{1 \text{ cm}^{-3}} \right) \left(\frac{T_{\infty}}{10^4 \text{ K}} \right)^{-3/2} \left(\frac{M}{M_{\odot}} \right)$$