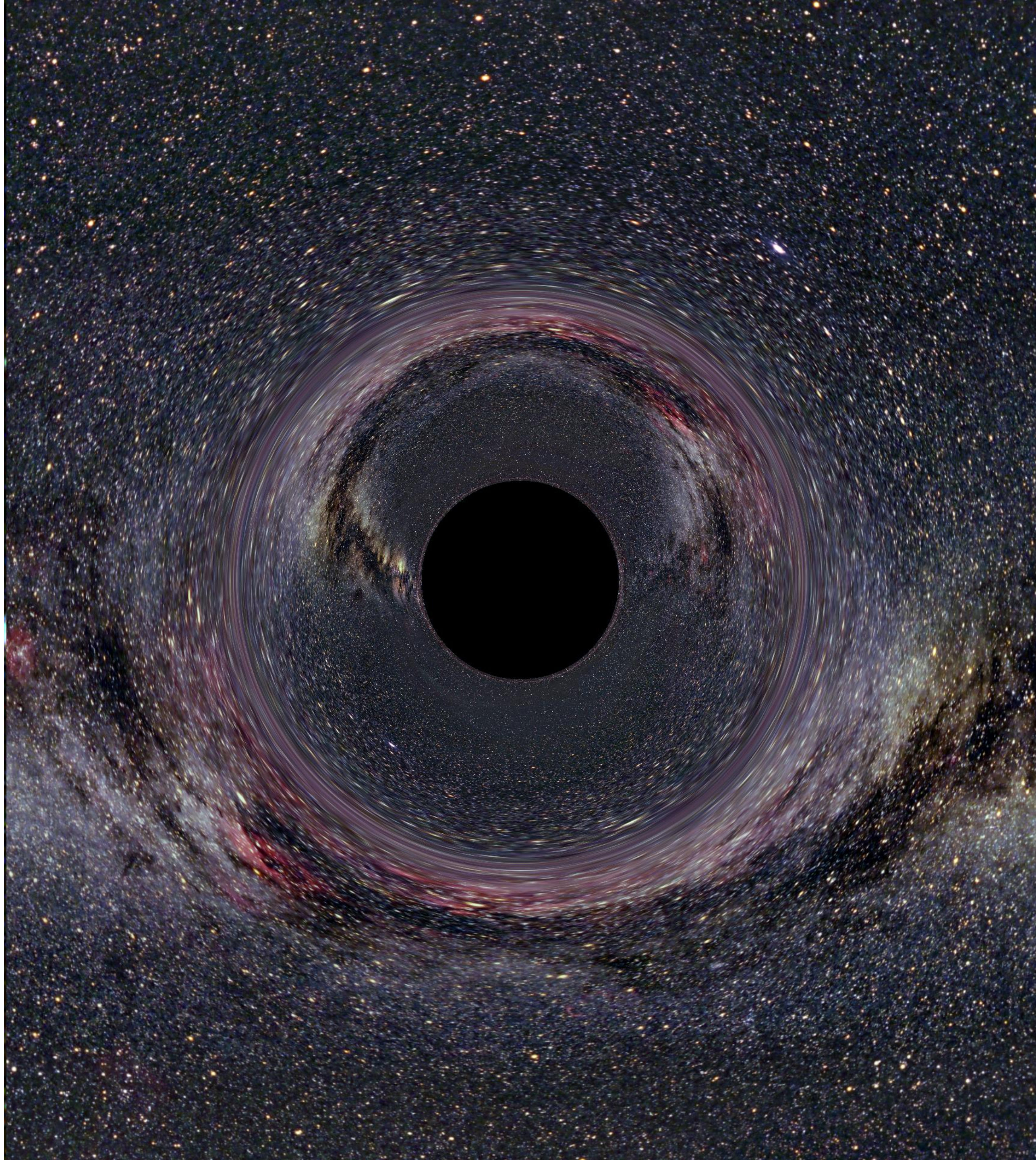
PHY 7097 | Modern Astrophysics

Lecture XII.

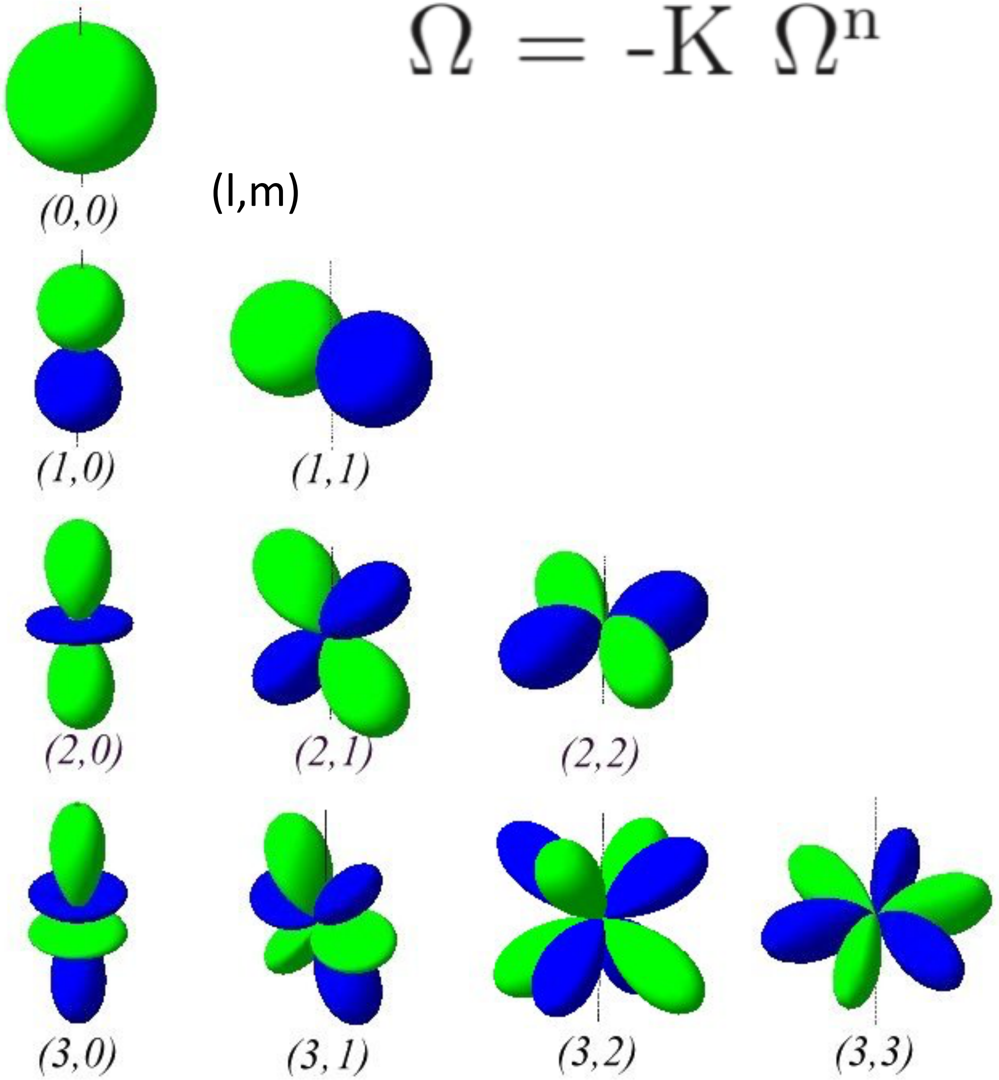
Black holes

Imre Bartos | Spring 2018



Pulsar breaking index (n)

$$\dot{\Omega} = -K\ \Omega^n$$



Relating multipole order and frequency derivatives:

In Jackson's Chapter 9, the time-average power radiated by an oscillating electric dipole $\boldsymbol{p}$, an oscillating quadrupole Q_{ij}, and a general ℓ-pole , are

$$P_1 = \frac{1}{3}\frac{\omega^4}{c^3}\frac{|\boldsymbol{p}|^2}{[4\pi\epsilon_0]}, \qquad P_2 = \frac{1}{360}\frac{\omega^6}{c^5}\frac{|Q_{ij}|^2}{[4\pi\epsilon_0]}, \qquad P_l = K\omega^{2\ell+2}.$$

(In gaussian units the factor in brackets is 1.) For gravitational radiation, the time-average quadrupole power is

$$P = \frac{1}{10}\frac{\omega^6}{c^5}G|I\!\!\!-_{ij}|^2,$$

where $I\!\!\!-_{ij}$ is the reduced (traceless) moment of inertia tensor (differs from the electromagnetic quadrupole by a factor of 3 in the definition of the moment, 9 in the power) and higher orders presumably follow the same pattern, although I have never looked at it.

If the energy radiated is taken from the kinetic energy of rotation,

$$P = \frac{d}{dt}\frac{1}{2}I\omega^2 = I\,\omega\,\dot{\omega},$$

then

$$I\omega\dot{\omega} = -K\omega^{2\ell+2}, \qquad \dot{\omega} = -\frac{K}{I}\,\omega^{2\ell+1},$$

and, differentiating,

$$\ddot{\omega} = -(2\ell+1)\,\frac{K}{I}\,\omega^{2\ell}.$$

Dividing,

$$\frac{\ddot{\omega}}{\dot{\omega}} = \frac{2\ell+1}{\omega}, \qquad n = \frac{\ddot{\omega}\,\omega}{\dot{\omega}^2} = 2\ell+1.$$

This last quantity is called the "braking index." For the Crab, this is observed to be $n = 2.5$, see `https://academic.oup.com/mnras/article/233/3/667/990298`

Formation mechanisms?

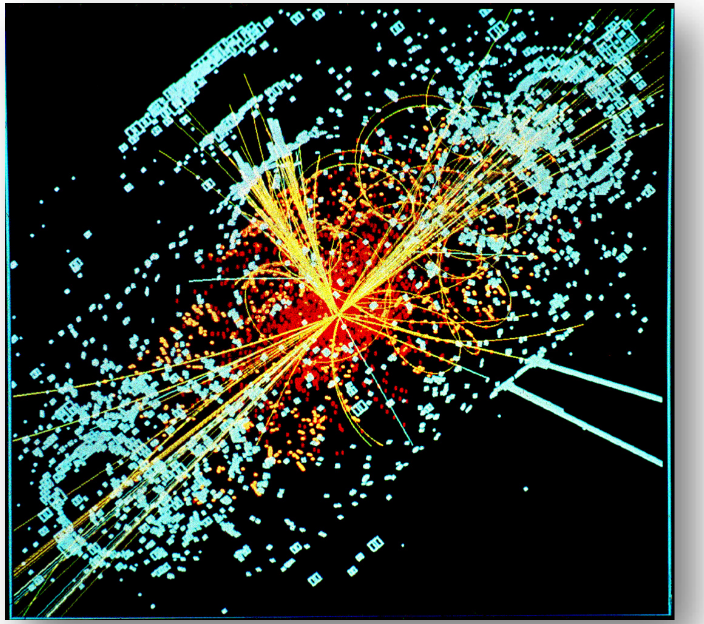
Micro-black holes

Schwarzschild radius: $R = 2GM/c^2$

Compton wavelength: $\lambda = h/Mc$

$$\left.\begin{array}{l} R = 2GM/c^2 \\ \lambda = h/Mc \end{array}\right\} M \geq Planck\ mass$$

(10^{-8} kg)



In some extensions of present physics (e.g. string theory) gravity can increase faster at short distances → lower minimum BH mass

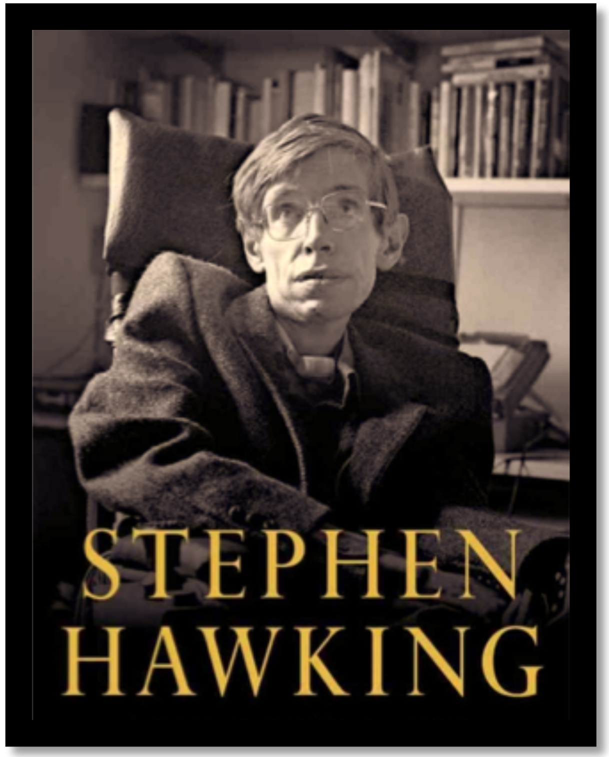
The Large Hadron Collider (LHC) and cosmic rays could produce BHs!

If such a small BH was created on Earth, would it be dangerous?

Hawking radiation

Black hole “temperature”:

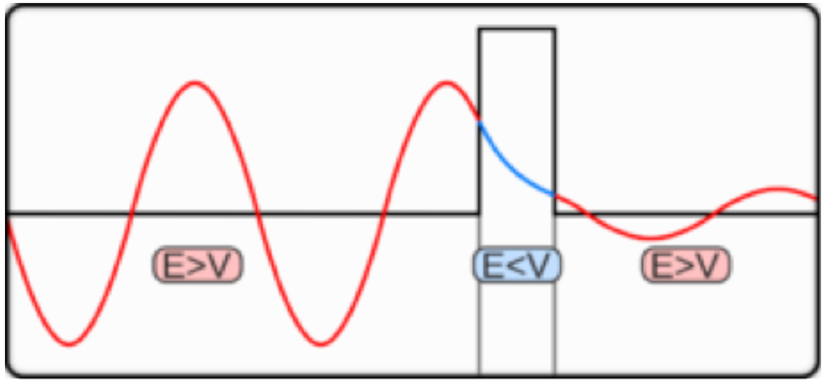
$$T = \frac{\hbar c^3}{8\pi G M k_B} \quad \left(\approx \frac{1.227 \times 10^{23}\ \mathrm{kg}}{M}\ \mathrm{K} = 6.169 \times 10^{-8}\ \mathrm{K} \times \frac{M_\odot}{M}\right)$$



Black hole will emit black body radiation at this temperature.

Irrelevant for astrophysical BHs.

Relevant for BH masses below 10^{12} kg.



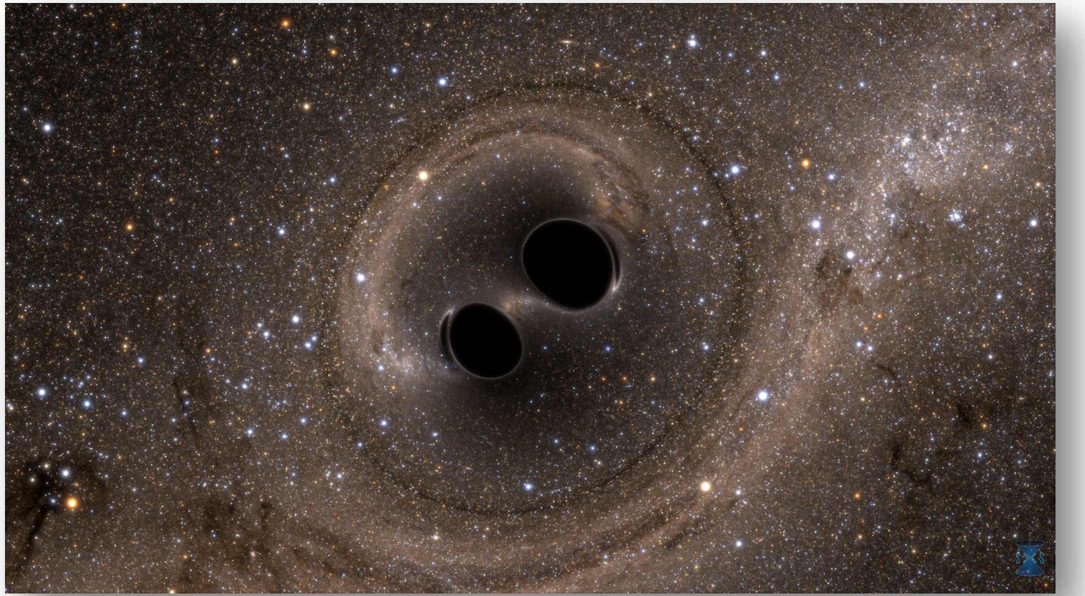
HW: What is the heaviest BH that, created on Earth, you would survive?

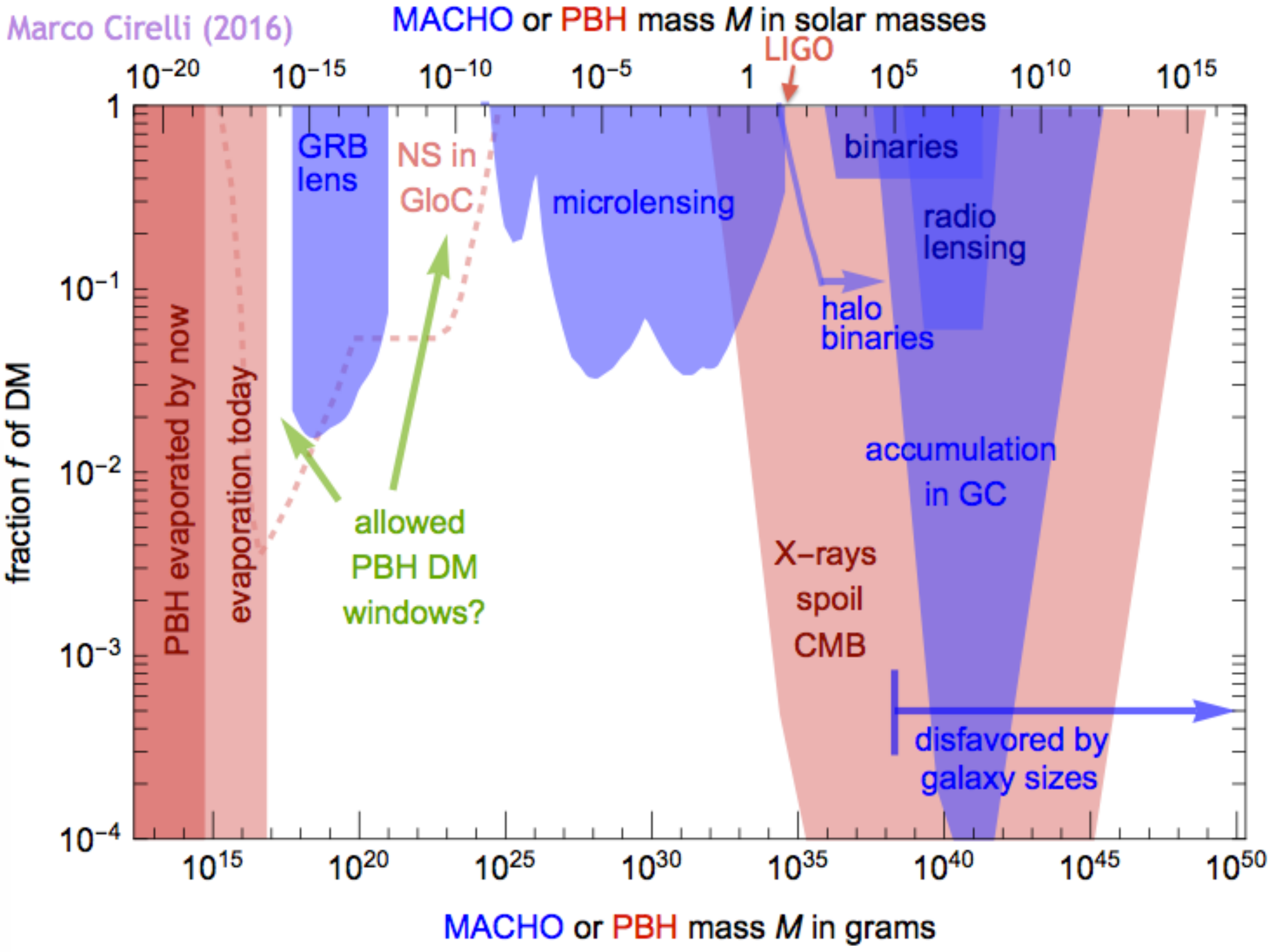
Primordial black holes

Large densities and density fluctuations soon after the big bang.

Mass distribution should be different from that of astrophysical black holes.

Primordial BHs were suggested as Dark Matter, and the origin of some LIGO BBH mergers, e.g. GW150914 (Bird+ 2016)





Black hole mass distribution

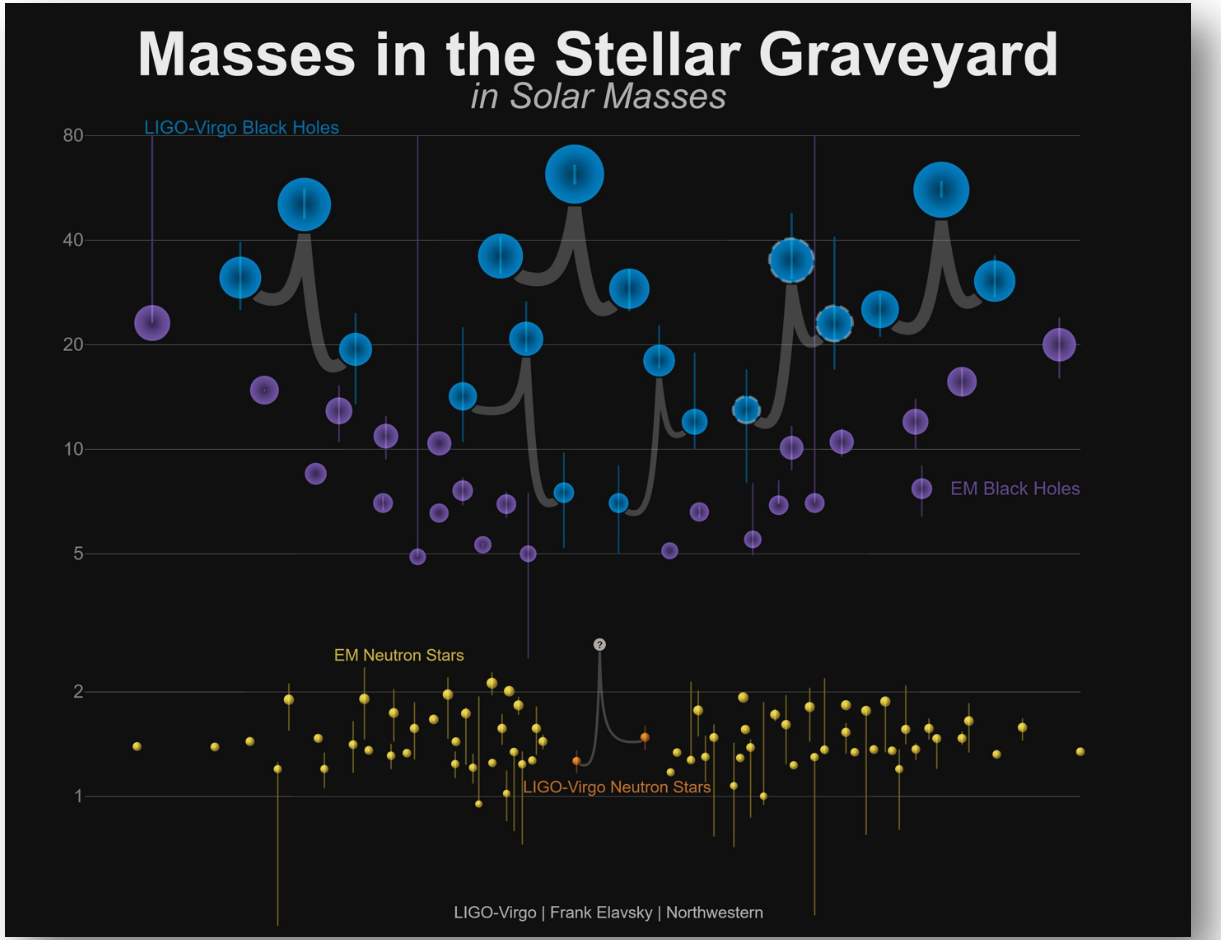
Three classes with different origin / evolution

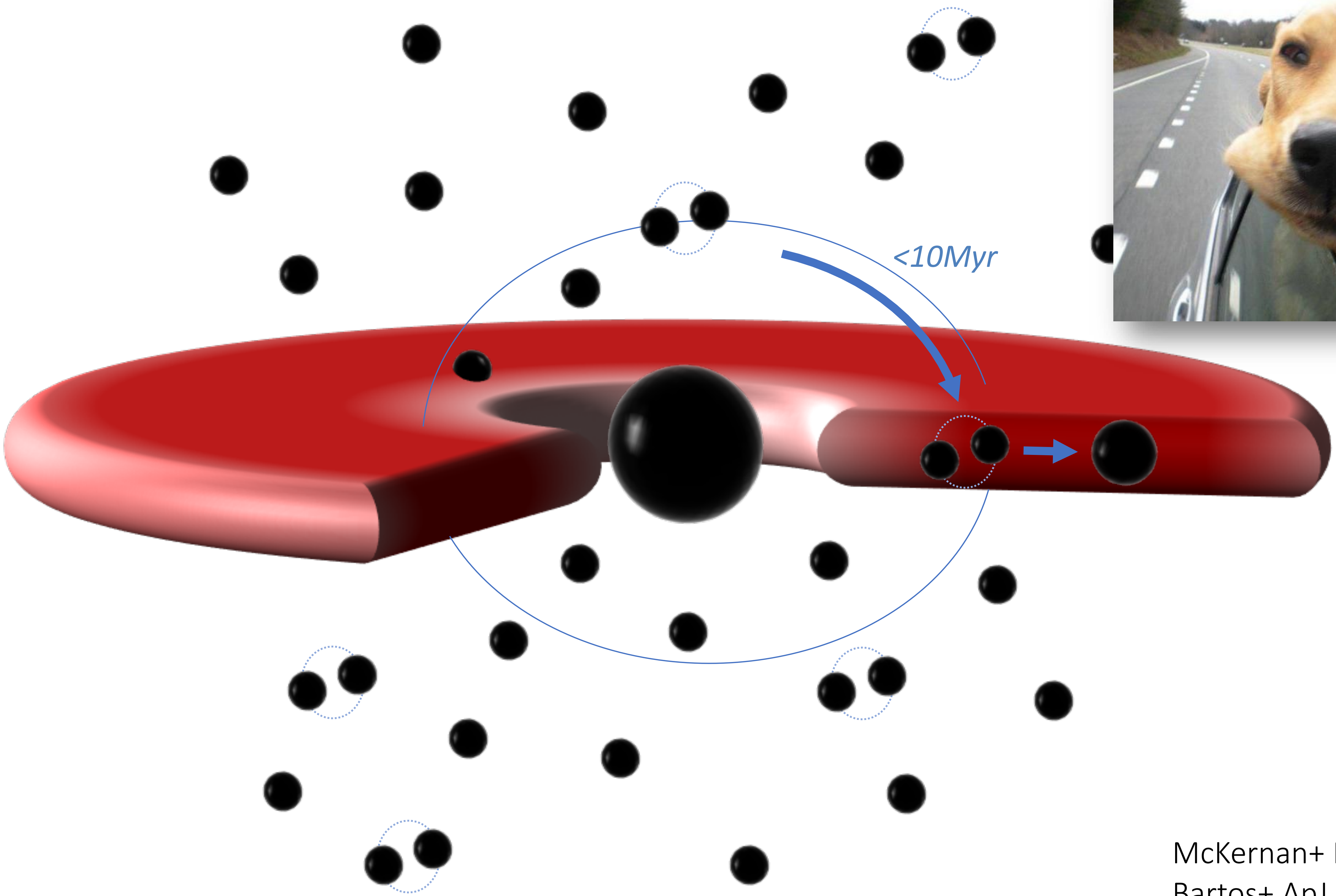
- Stellar-mass (3 Msun – ~100 Msun)
- Intermediate-mass (~100 Msun – 10^5 Msun)
- Supermassive (10^5 Msun – 10^{11} Msun)

Stellar mass mass distribution:

- We don't know
- There seems to be a mass lower limit at 5 Msun
  → mass gap
- Best guess – Salpeter function (PDF ~ $M^{-2.3}$)

LIGO's detected BH mass distribution is consistent with Salpeter function up to a cutoff mass of ~ 50 Msun





McKernan+ MNRAS 2012
Bartos+ ApJ 2017

Intermediate mass black holes

No confirmed observation.

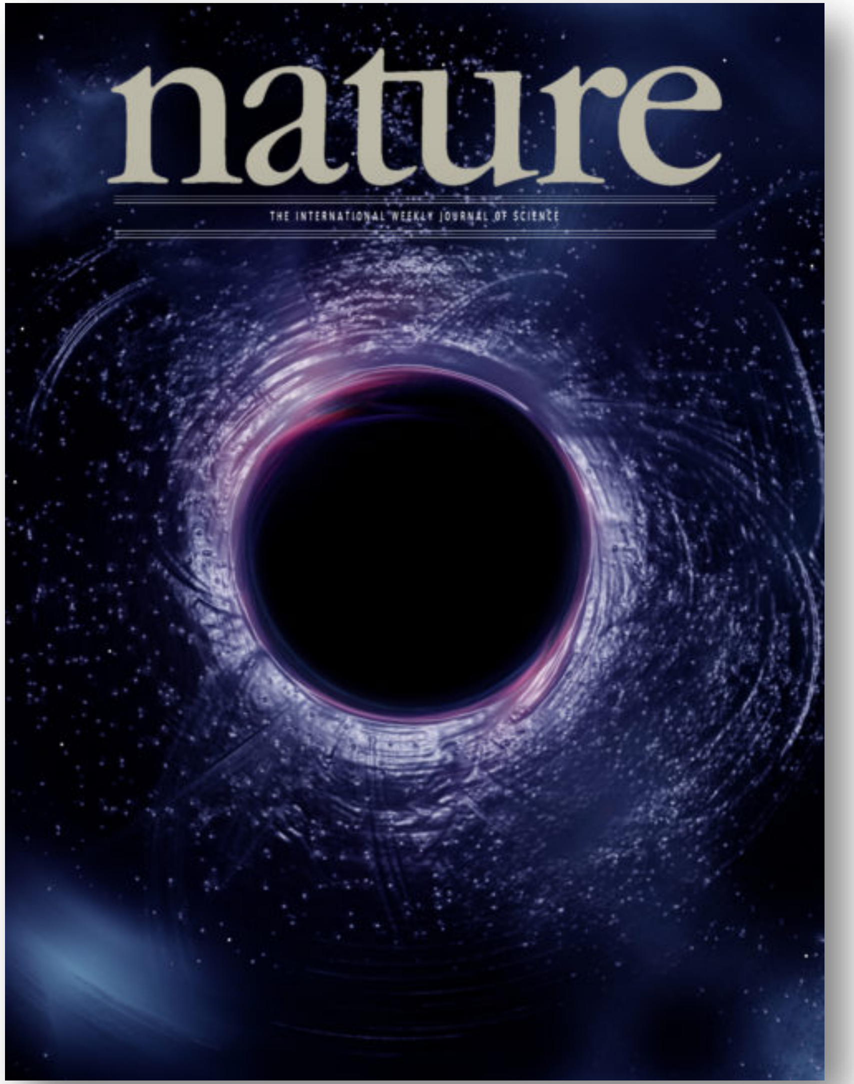
Claims:

- Measurement of Doppler shift of stellar radiation in X-ray binaries.
- Super-Eddington radiation in X-ray binaries.
- Stellar dynamics in globular clusters.
- …

LIGO has limits on their abundance.

Origin:

- From stellar mass BHs through accretion
- Collision of multiple stars or stellar remnants in dense environments
- Primordial BHs
- Collapse of Pop III stars



Supermassive black holes

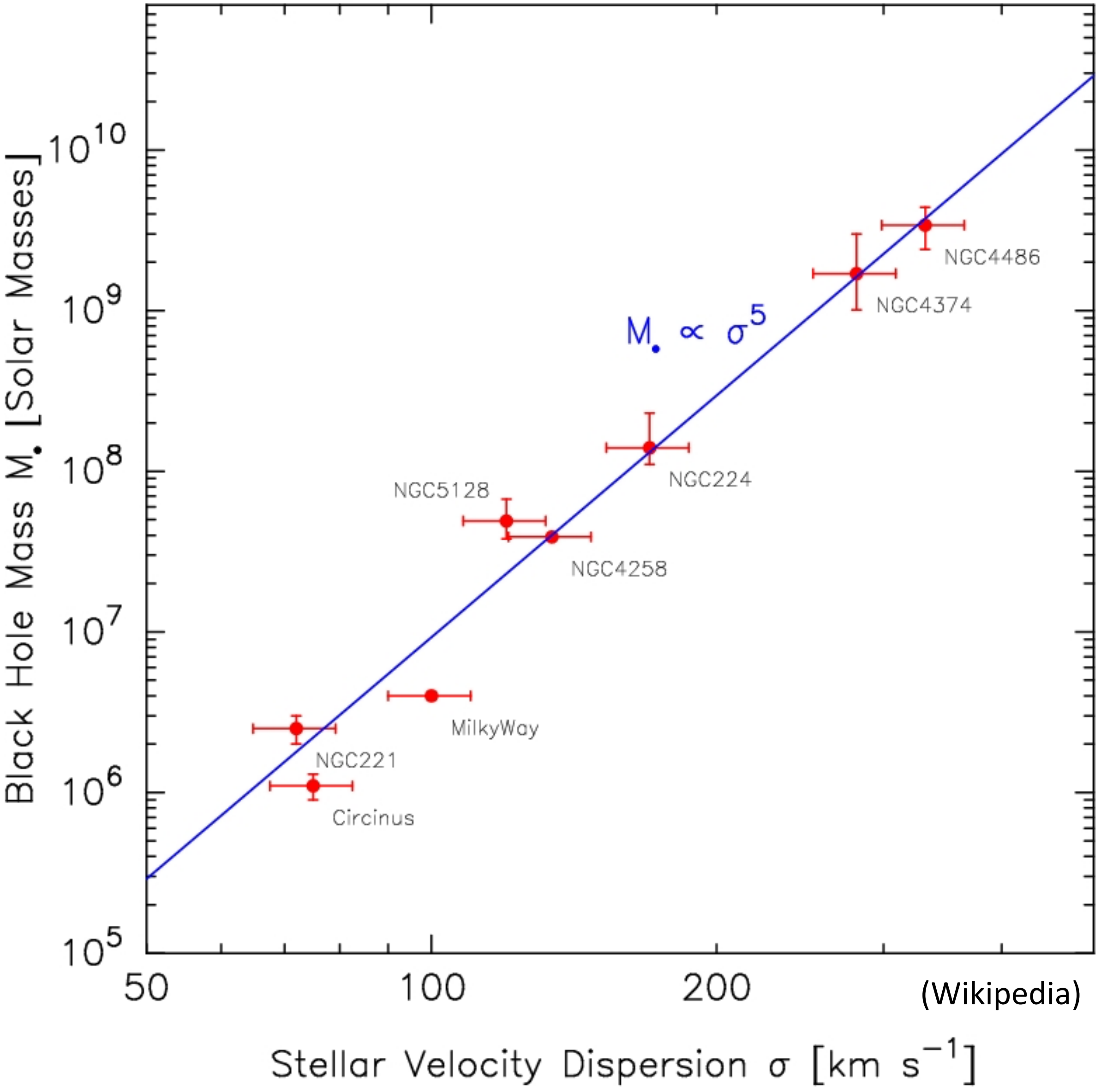
<u>Formation:</u> needs a seed
- Very massive star collapses
- Primordial black hole

But we don't know.

<u>Growth:</u>
- Accretion
- Merger with other black holes

But we don't really know.

They are fundamental components of galaxies:
- M-sigma relation.

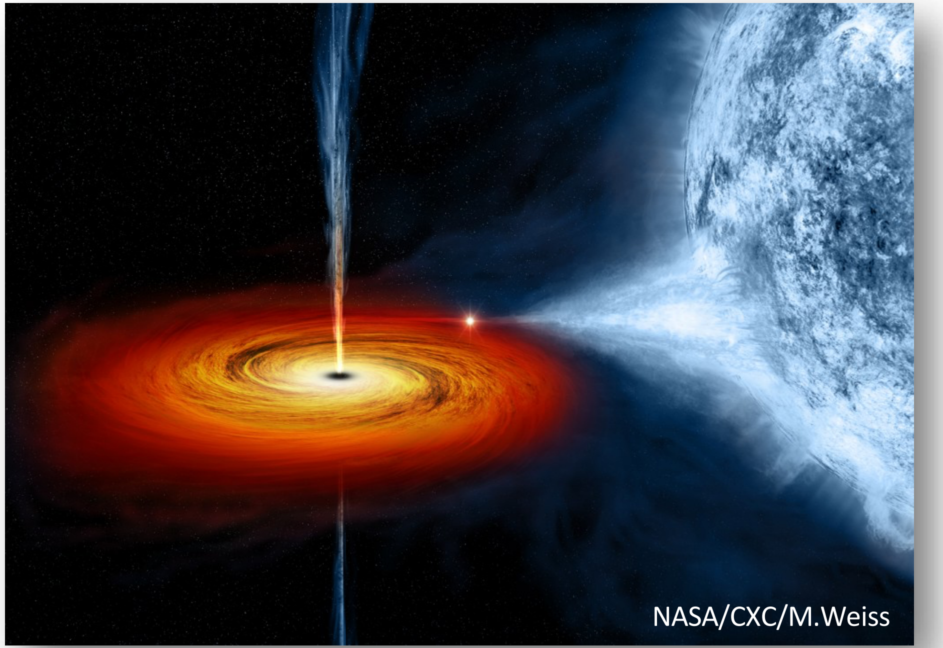


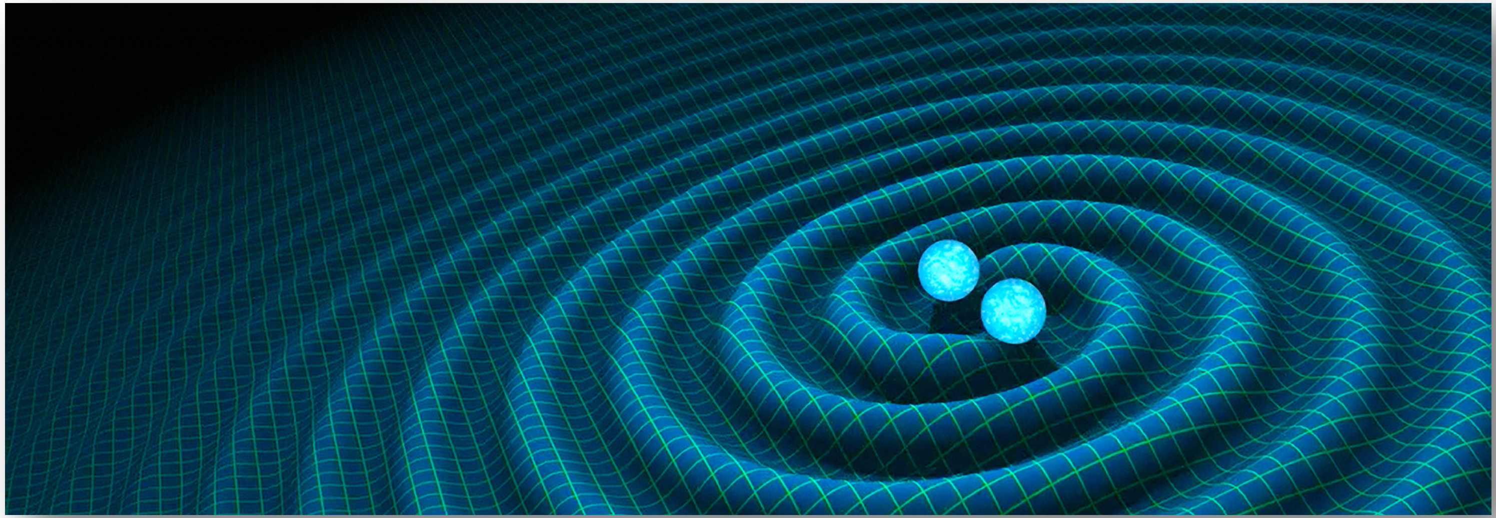
(Wikipedia)

Observations

Hawking radiation is undetectable.

- Gravitational waves!
- Motion of stars orbiting Sagittarius A*
- Accretion of matter
- Gravitational lensing (not yet observed)





Event Horizon Telescope

- Network of radio telescopes.
- Interferometry
- Observe the immediate environment of the SMBH in the Milky Way (Sagittarius A*); and maybe others.
- Can resolve distance on the order of the Schwarzschild radius.
- Results expected soon…