PHY 6646 - Quantum Mechanics II - Spring 2020
Homework set # 6, due February 19

1. Problems 16.1.2 and 16.1.3 in Shankar's book.

2. Consider a particle of mass μ in one dimensional motion in the potential:

$$\begin{aligned} V(x) &= eEx \qquad \text{for } x > 0 \\ &= \infty \qquad \text{for } x < 0 \quad . \end{aligned} \tag{0.1}$$

This potential is called a *triangular barrier*, for obvious reasons. It is the potential seen by an electron in a uniform electric field for $x > 0$ and a 'hard wall' at $x = 0$. It is relevant to semiconductor device physics where $V(x)$ is a good approximation to the potential in a Si MOSFET at the Si/SiO_2 interface. Electrons trapped at the interface behave in many ways like a two-dimensional electron gas and many interesting effects such as the Quantum Hall Effect can occur in these devices. Use the trial wavefunction $\Psi(x) = xe^{-\alpha x}$ for $x > 0$ and 0 otherwise, with α real. This wavefunction insures that the boundary condition $\Psi(0) = 0$ is obeyed. Solve for the variational parameter α and estimate the energy of the ground state.

3. The variational technique can sometimes be used to find excited states.

a. For the hydrogen atom, show that the minimum expectation value of the energy in the state described by the wavefunction $\Psi(\vec{r}) = r\ \sin\theta\ e^{i\phi}\ f(r)$ is - 0.25 Ry.

b. Minimize the energy with respect to α for wavefunctions of the form $r\ \sin\theta\ e^{i\phi}\ e^{-\alpha r^2}$, and compare your result to the minimum above.

c. What would happen if you used the variational function $r\ \sin\theta\ e^{i\phi}\ e^{-\alpha r}$ instead?

d. How would you use the variational method to get the higher lying excited states of the hydrogen atom?

4. Problem 16.2.4 in Shankar's book.