PHY 6646 - Quantum Mechanics II - Spring 2020
Homework #11, due April 1

1. The four lowest energy states of the hydrogen atom are split in energy by the interaction between electron spin and proton spin. See problem 15.1.2. The groundstate has total spin $s = 0$. At an energy $\hbar\omega_0$, where $\omega_0 = (2\pi)$ 1.42 GHz, above the ground state are three states with $s = 1$. The electromagnetic radiation emitted or absorbed by transitions between the $s = 0$ and $s = 1$ states is commonly referred to as the “21 cm line”

a) A plane electromagnetic wave of wavevector $\vec{k}$, frequency $\omega = ck$, and energy per unit surface and unit time I, is incident upon a hydrogen atom in its ground state. The electromagnetic wave causes a time-dependent perturbation of the hydrogen atom

$$H^1(t) = -\gamma_e \vec{S}_e \cdot \vec{B}(\vec{0}, t) \tag{0.1}$$

where $\vec{S}_e$ is the spin of the electron, γ_e the electron gyromagnetic ratio, and $\vec{B}(\vec{0}, t)$ the magnetic field at the location of the atom. Eq. (0.1) neglects the much weaker interaction ($\gamma_p << \gamma_e$) of the electromagnetic field with the proton spin. Obtain the rate of transition of the atom from its $s = 0$ ground state to any of the three $s = 1$ states as a result of this perturbation. Does the rate depend on the direction of $\vec{k}$? Does it depend on the state of polarization of the electromagnetic wave?

b) The atom in its ground state is placed in a thermal bath of electromagnetic radiation at temperature T. Show that the rate at which it makes transitions to the $s = 1$ states is given by

$$R = \left(\frac{e}{m_e c}\right)^2 \frac{\hbar\omega_0^3}{c^3} \frac{1}{e^{\hbar\omega_0/k_b T} - 1} \tag{0.2}$$

where m_e is the electron mass, and k_B is Boltzmann’s constant.

c) Verify that Eq. 0.2 is dimensionally correct.

2. Problems 21.1.2, 21.1.3 and 21.1.16 in Shankar’s book.