

**PHY 6646 - Quantum Mechanics II - Spring 2018**  
**Homework #13, due April 18**

1. Consider the properties of a fluid composed of a huge number  $N$  (of order Avogadro's number) of identical spinless particles moving in a given potential  $V(\vec{r}, t)$  and all in the same state. The particles do not interact with one another. The state is described by a wavefunction  $\Psi(\vec{r}, t)$  which is a solution of the time-dependent Schrödinger equation

$$i\partial_t\Psi = \left(-\frac{\hbar^2}{2\mu}\nabla^2 + V(\vec{r}, t)\right)\Psi \quad . \quad (0.1)$$

The wavefunction may be written in terms of a real amplitude and a phase:  $\Psi(\vec{r}, t) = A(\vec{r}, t)e^{i\beta(\vec{r}, t)}$ . Assume it has unit norm.

a) Justify the following expression for the density of the fluid

$$n(\vec{r}, t) = NA(\vec{r}, t)^2 \quad . \quad (0.2)$$

b) Motivate the following expression for the velocity field of the fluid

$$\vec{v}(\vec{r}, t) = \frac{\hbar}{\mu}\vec{\nabla}\beta(\vec{r}, t) \quad . \quad (0.3)$$

c) Using Eq. (0.1), show that the continuity equation is satisfied.

d) Also using Eq. (0.1), derive the following Euler-like equation for the velocity field:

$$\partial_t\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\mu}\vec{\nabla}V - \vec{\nabla}q \quad , \quad (0.4)$$

where

$$q(\vec{r}, t) = -\frac{\hbar^2}{2\mu^2}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} \quad . \quad (0.5)$$

How does Eq. (0.4) differ from the corresponding equation for classical particles?

2. Problem 19.5.5

3. Problem 21.1.16

4. Problem 21.2.2