PHY 6645 - Quantum Mechanics I - Fall 2018
Homework #7, due October 10

1. Consider the properties of a fluid composed of a huge number N (of order Avogadro's number) of identical spinless particles moving in a given potential $V(\vec{r},t)$ and all in the same state. The particles do not interact with one another. The state is described by a wavefunction $\Psi(\vec{r},t)$ which is a solution of the time-dependent Schrödinger equation

$$i\partial_t \Psi = (-\frac{\hbar^2}{2\mu}\nabla^2 + V(\vec{r},t))\Psi \quad . \tag{0.1}$$

The wavefunction may be written in terms of a real amplitude and a phase: $\Psi(\vec{r},t) = A(\vec{r},t)e^{i\beta(\vec{r},t)}$. Assume it has unit norm.

a) Justify the following expression for the density of the fluid

$$n(\vec{r},t) = NA(\vec{r},t)^2 \quad . \tag{0.2}$$

b) Motivate the following expression for the velocity field of the fluid

$$\vec{v}(\vec{r},t) = \frac{\hbar}{\mu}\vec{\nabla}\beta(\vec{r},t) \quad . \tag{0.3}$$

c) Using Eq. (0.1), show that the continuity equation is satisfied.

d) Also using Eq. (0.1), derive the following Euler-like equation for the velocity field:

$$\partial_t \vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\mu}\vec{\nabla}V - \vec{\nabla}q \quad , \tag{0.4}$$

where

$$q(\vec{r},t) = -\frac{\hbar^2}{2\mu^2}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} \quad . \tag{0.5}$$

How does Eq. (0.4) differ from the corresponding equation for classical particles?

2. Problems 7.3.1, 7.3.2, 7.4.7 and 7.4.9