PHY 6645 - Quantum Mechanics I - Fall 2018
Homework set # 6, due October 3

1. Let $H(\lambda)$ be the Hamiltonian of a system which depends explcitly upon a parameter λ. Consider the eigenvalues $E_n(\lambda)$ and eigenstates $|E_n(\lambda)>$ of $H(\lambda)$. Show that

$$< E_n(\lambda)|\frac{dH}{d\lambda}|E_n(\lambda)>= \frac{dE_n}{d\lambda} \quad , \tag{0.1}$$

for all n.

2. a) Consider two linear operators A and B. Show that

$$e^A B e^{-A} = \sum_{n=0}^{\infty} \frac{1}{n!}[A,[A,...[A,B]...]] \tag{0.2}$$

where $[A,[A,...[A,B]...]] \equiv [A,[A,[A,B]]]$ for $n=3$, and so on.

b) Show that

$$\begin{aligned} e^{\frac{i}{\hbar}x_0 P} F(X,P) e^{-\frac{i}{\hbar}x_0 P} &= F(X+x_0,P) \\ e^{-\frac{i}{\hbar}p_0 X} F(X,P) e^{\frac{i}{\hbar}p_0 X} &= F(X,P+p_0) \end{aligned} \tag{0.3}$$

where X and P are canonically conjugate operators ($[X,P]=i\hbar$) and x_0 and p_0 are c-numbers.

3. A particle of mass μ and energy E is approaching a sudden potential drop V_0. The potential is

$$\begin{aligned} V(x) &= 0 \quad \text{for } x<0 \\ &= -V_0 \quad \text{for } x>0 \quad . \end{aligned} \tag{0.4}$$

The particle comes in from the $x<0$ region.

a) What are the probabilities of reflection and transmission?

b) Does a classical particle get reflected under such circumstances? Does the reflection probability of the quantum mechanical particle go to zero as $\hbar \to 0$? Why not? When do actual particles get reflected by potential drops?

4. Problem 5.4.3 in Shankar's book.