10/2/18
R.P. Woodard

Solutions to Asmt #8

(1a) * $x(t) = 10ct$

(1b) * $\begin{Bmatrix} ct' = \gamma(ct - \beta x) \\ x' = \gamma(x - \beta ct) \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} ct = \gamma(ct' + \beta x') \\ x = \gamma(x' + \beta ct') \end{Bmatrix}$ & $\begin{Bmatrix} \beta = \frac{4}{5} \\ \gamma = 5/3 \end{Bmatrix} \to \begin{Bmatrix} ct' = -\frac{35}{3} ct \\ x' = +\frac{46}{3} ct \end{Bmatrix} \to \boxed{x'(t') = \frac{-46}{35} ct'}$

(1c) * turn-around at $t = T \to (ct', x') \equiv (cT', X') = \left(-\frac{35}{3} cT, \frac{46}{3} cT\right)$

* world line is $\boxed{x'(t') = X' - 10c(t' - T') = \frac{-304}{3} cT - 10ct'}$

(1d) * $ct = \frac{5}{3}\left[ct' + \frac{4}{5}\left(-\frac{304}{3} cT - 10ct'\right)\right] = -\frac{1216}{9} cT - \frac{35}{3} ct'$ (NB increasing t' makes t smaller)

* $x = \frac{5}{3}\left[-\frac{304}{3} cT - 10ct' + \frac{4}{5} ct'\right] = -\frac{1520}{9} cT - \frac{46}{3} ct' \to x(t) = \frac{304}{35} cT + \frac{46}{35} ct$

(1e) * $x = 0$ at $t = \frac{-152}{23} T \doteq -6.61 T$

(1f) * $t = -12$ hrs $\to T = \frac{69}{38}$ hrs $\doteq$ 1h 48m 56.8s

(2a) * $p^\mu = (Mc, \vec{0}) = (\gamma mc, \gamma mc\vec{\beta}) + (\gamma mc, -\gamma mc\vec{\beta}) = \overset{,\vec{0}}{(2\gamma mc)} \to m = \frac{M}{2\gamma} = \frac{3}{10} M$

(2b) * $0.48 = \frac{1}{2\gamma} \to \sqrt{1-\beta^2} = 0.96 \to \beta = 0.28 \to V = 0.28c$

(2c) * now $p^\mu = (Mc, \vec{0}) = (3\gamma mc, \vec{0}) \to 0.32 = \frac{1}{3\gamma} \to \sqrt{1-\beta^2} = 0.96 \to V = 0.28c$

(3a) * $\frac{\partial L}{\partial \dot{x}} = \frac{mc\dot{x}}{\sqrt{c^2 - \dot{x}^2}}$
* $\frac{\partial L}{\partial x} = ma$ $\Big\} \to \frac{d}{dt}\left(\frac{mc\dot{x}}{\sqrt{c^2-\dot{x}^2}}\right) = ma$

(3b) * integrating once $\to \frac{c\dot{x}(t)}{\sqrt{c^2 - \dot{x}^2(t)}} - \frac{c\dot{x}_0}{\sqrt{c^2 - \dot{x}_0^2}} = at \to \dot{x}(t) = \frac{c\left(\frac{\dot{x}_0}{\sqrt{c^2-\dot{x}_0^2}} + \frac{a}{c}t\right)}{\sqrt{1 + \left(\frac{\dot{x}_0}{\sqrt{c^2-\dot{x}_0^2}} + \frac{a}{c}t\right)^2}}$

* integrating again $\to x(t) = x_0 + \frac{c^2}{a}\left[\sqrt{1 + \left(\frac{\dot{x}_0}{\sqrt{c^2-\dot{x}_0^2}} + \frac{a}{c}t\right)^2} - \frac{1}{\sqrt{1 - \dot{x}_0^2/c^2}}\right]$

(3c) * $x_0 = 0 = \dot{x}_0 \to x(t) = \frac{c^2}{a}\left[\sqrt{1 + \left(\frac{at}{c}\right)^2} - 1\right]$

* photon released at $t = t_0$ has $x_\gamma(t) = c(t - t_0)$

* $x(t) = x_\gamma(t)$ at same time $t \to t = \left(\frac{1 - \frac{at_0}{2c}}{1 - \frac{at_0}{c}}\right) t_0$

* for t to be finite we need $1 - \frac{at_0}{c} > 0 \to t_0 < c/a$