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Solutions to Asmt #6

(1a) * $\vec{\omega} = \omega\,[\sin(\theta)\,\hat{y} + \cos(\theta)\,\hat{z}]$ $\quad \omega = \frac{2\pi}{T} \quad T = 24\text{ hrs}$

ignore non-horizontal component

(1b) * $-2\vec{\omega}\times\vec{v} = -2\omega\,[-\cos(\theta)\,v_y\,\hat{x} + \cos(\theta)\,v_x\,\hat{y} - \sin(\theta)\,v_x\,\hat{z}]$

$$\therefore \begin{Bmatrix} \dot{v}_x = +2\omega\cos\theta\, v_y \\ \dot{v}_y = -2\omega\cos\theta\, v_x \end{Bmatrix} \rightarrow \begin{Bmatrix} \ddot{v}_x = 2\omega\cos\theta\,\dot{v}_y = -4\cos^2\theta\,\omega^2 v_x \\ \ddot{v}_y = -2\omega\cos\theta\,\dot{v}_x = -4\cos^2\theta\,\omega^2 v_y \end{Bmatrix}$$

(1c) * NB $\dot{v}_x(0) = 2\cos\theta\,\omega\, v_y(0)$ & $\dot{v}_y(0) = -2\cos\theta\,\omega\, v_x(0)$

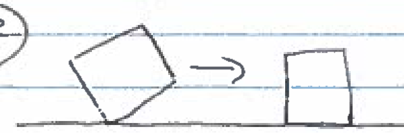
$$\rightarrow \begin{Bmatrix} v_x(t) = v_x(0)\cos[2\cos\theta\,\omega t] + v_y(0)\sin[2\cos\theta\,\omega t] \\ v_y(t) = -v_x(0)\sin[2\cos\theta\,\omega t] + v_y(0)\cos[2\cos\theta\,\omega t] \end{Bmatrix}$$

(1d) * $\theta = 60^\circ \rightarrow 2\cos(\theta) = 1$

* $v_y(0) = 0 \rightarrow \begin{Bmatrix} v_x(t) = v\cos(\omega t) \\ v_y(t) = -v\sin(\omega t) \end{Bmatrix}$ moves south at $\omega t = \frac{\pi}{2} \rightarrow t = 6\text{ hrs}$

(2a) * $I_{center} = 8L\rho_0 \int_0^{\pi/4} d\phi \int_0^{\frac{1}{2}L\sec(\phi)} d\rho\,\rho \times \rho^2 = \frac{1}{6} M L^2$ $\quad \rho_0 = \frac{M}{L^3}$

* Parallel axis thm $\rightarrow I_{edge} = M\left(\frac{L}{\sqrt{2}}\right)^2 + I_{center} = \frac{2}{3} M L^2$



(2b) * $E = \frac{1}{2} I_{edge}\,\omega^2 + Mgz$ ← com position

$$\begin{Bmatrix} \text{* Initial: } \omega = 0 \ \&\ z = \frac{L}{\sqrt{2}} \\ \text{* Final: } z = \frac{L}{2} \end{Bmatrix} \rightarrow \frac{1}{2} I_{edge}\,\omega^2 = \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) M g L$$

(2c) * $\omega = \sqrt{3\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) g/L}$

(3a) * $L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - \frac{1}{2} k_1 x_1^2 - \frac{1}{2} k_2 (x_1 - x_2)^2 - \frac{1}{2} k_3 x_2^2$

$$\rightarrow \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} = -\begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \Omega^2 = \begin{pmatrix} \frac{k_1 + k_2}{m_1} & -\frac{k_2}{m_1} \\ -\frac{k_2}{m_2} & \frac{k_2 + k_3}{m_2} \end{pmatrix}$$

(3b) $\begin{Bmatrix} * m_i = i m \\ * k_i = i k \end{Bmatrix} \rightarrow \Omega^2 = \frac{k}{m}\begin{pmatrix} 3 & -2 \\ -1 & 5/2 \end{pmatrix} \rightarrow \omega_\pm^2 = \frac{1}{4}\frac{k}{m}\left(11 \pm \sqrt{33}\right)$

(3c) * $3x - 2y = \frac{1}{4}(11 \pm \sqrt{33})\, x \rightarrow y = \frac{1}{8}(1 \mp \sqrt{33})\, x$