11/24/18
R.P. Woodard

Solutions to Asmt #11

(1a) * $g_{00} = -B(r)$ & $g_{ij} = A(r)\delta_{ij}$

* $\Gamma^{\rho}{}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(g_{\sigma\mu,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma})$

* $\Gamma^{0}{}_{0i} = \frac{B'}{2B}\hat{r}^i$, $\Gamma^{i}{}_{00} = \frac{B'}{2A}\hat{r}^i$ & $\Gamma^{i}{}_{jk} = \frac{A'}{2A}(\delta_{ij}\hat{r}_k + \delta_{ki}\hat{r}_j - \delta_{jk}\hat{r}_i)$

(1b) * $R^{\rho}{}_{\sigma\mu\nu} \equiv \Gamma^{\rho}{}_{\nu\sigma,\mu} - \Gamma^{\rho}{}_{\mu\sigma,\nu} + \Gamma^{\rho}{}_{\mu\alpha}\Gamma^{\alpha}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\alpha}\Gamma^{\alpha}{}_{\mu\sigma}$

* $R^{0}{}_{i0j} = \left(-\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{2AB} + \frac{B'}{2rB}\right)\hat{r}_i\hat{r}_j - \left(\frac{A'B'}{4AB} + \frac{B'}{2rB}\right)\delta_{ij}$

* $R^{i}{}_{jk\ell} = \left(-\frac{A''}{2A} + \frac{3A'^2}{4A^2} + \frac{A'}{2rA}\right)(\delta_{ik}\hat{r}_j\hat{r}_\ell - \delta_{kj}\hat{r}_\ell\hat{r}_i + \delta_{j\ell}\hat{r}_i\hat{r}_k - \delta_{\ell i}\hat{r}_k\hat{r}_j) - \left(\frac{A'^2}{4A^2} + \frac{A'}{rA}\right)(\delta_{ik}\delta_{j\ell} - \delta_{i\ell}\delta_{jk})$

(1c) * $R_{\mu\nu} \equiv R^{\rho}{}_{\mu\rho\nu}$

* $R_{00} = \frac{B}{A}\left(\frac{B''}{2B} - \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{B'}{rB}\right)$

* $R_{ij} = \left(-\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{2AB} + \frac{B'}{2rB} - \frac{A''}{2A} + \frac{3A'^2}{4A^2} + \frac{A'}{2rA}\right)\hat{r}_i\hat{r}_j - \left(\frac{A'B'}{4AB} + \frac{B'}{2rB} + \frac{A''}{2A} - \frac{A'^2}{4A^2} + \frac{3A'}{2rA}\right)\delta_{ij}$

(1d) * $R \equiv g^{\mu\nu}R_{\mu\nu}$

$\rightarrow R = \frac{1}{A}\left[-\frac{B''}{B} + \frac{B'^2}{2B^2} - \frac{A'B'}{2AB} - \frac{2B'}{rB} - \frac{2A''}{A} + \frac{3A'^2}{2A^2} - \frac{4A'}{rA}\right]$

(1e) * $A = [1 + \frac{K}{r}]^4 \rightarrow \frac{A'}{A} = \frac{-4K/r^2}{1+K/r}$ & $\frac{A''}{A} = \frac{8K/r^3}{1+K/r} + 12\left(\frac{K/r^2}{1+K/r}\right)^2 = \frac{3}{4}\frac{A'^2}{A^2} - \frac{2A'}{rA}$

* $B = \frac{1}{A}(1 - K^2/r^2)^2 \rightarrow \frac{B'}{B} = -\frac{A'}{A} + \frac{4K^2/r^3}{1-K^2/r^2} = \frac{4K/r^2}{1-K^2/r^2}$ & $\frac{B''}{B} - \frac{B'^2}{2B^2} = \frac{-8K/r^3}{(1+\frac{K}{r})^2(1-\frac{K}{r})} = -\frac{A'B'}{2AB} - \frac{2B'}{rB}$

$\boxed{\text{3 coefficients}}$

* $\alpha \equiv \frac{B''}{2B} - \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{B'}{rB} = 0$ by the relation for $\frac{B''}{B} - \frac{B'^2}{2B^2}$ $\rightarrow R_{00} = 0$

* $\beta \equiv -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{2AB} + \frac{B'}{2rB} - \frac{A''}{2A} + \frac{3A'^2}{4A^2} + \frac{A'}{2rA} \xrightarrow{A'' \,\&\, B''} \frac{3}{2}\left[\frac{A'B'}{2AB} + \frac{B'}{rB} + \frac{A'^2}{4A^2} + \frac{A'}{rA}\right]$

* $\gamma \equiv \frac{A'B'}{4AB} + \frac{B'}{2rB} + \frac{A''}{2A} - \frac{A'^2}{4A^2} + \frac{3A'}{2rA} \xrightarrow{A''} \frac{1}{2}\left[\frac{A'B'}{2AB} + \frac{B'}{rB} + \frac{A'^2}{4A^2} + \frac{A'}{rA}\right]$

* NB $\frac{A'B'}{2AB} + \frac{B'}{rB} = \frac{+4K/r^3}{(1+K/r)^2} = -\frac{A'^2}{4A^2} - \frac{A'}{rA}$ $\rightarrow R_{ij} = 0$

(2a) * $\frac{\delta S}{\delta A_{a\mu}(x)} = \int d^4y \left[-\frac{1}{2}\frac{\delta F_{b\rho\sigma}(y)}{\delta A_{a\mu}(x)} F_b{}^{\rho\sigma}(y)\right] = \partial_\rho F_a{}^{\rho\mu}(x) - g f_{abc} A_{b\rho}(x) F_c{}^{\rho\mu}(x) = 0$

(2b) * $\mathcal{L} = \frac{1}{2}F_{a0i}F_{a0i} - \frac{1}{4}F_{aij}F_{aij} \xrightarrow[\text{Gauge}]{\text{Temporal}} \frac{1}{2}\dot{A}_{ai}\dot{A}_{ai} - \frac{1}{4}F_{aij}F_{aij}$

* $\Pi_a{}^i \equiv \frac{\partial\mathcal{L}}{\partial\dot{A}_{ai}} = \dot{A}_{ai} \rightarrow \mathcal{H} = \frac{1}{2}\Pi_a{}^i\Pi_a{}^i + \frac{1}{4}F_{aij}F_{aij}$ & $H = \int d^3x\, \mathcal{H}$

(2c) * Constraint is the $\mu = 0$ Euler-Lagrange Eqn $\rightarrow -\partial_i\Pi_a{}^i + g f_{abc} A_{bi}\Pi_c{}^i = 0$

(3a) * $\frac{\delta S}{\delta\phi(x)} = \partial^2\phi(x) - m^2\phi(x) - \lambda\phi^3(x) = 0$

* $\lambda = 0 \rightarrow \partial^2\phi(x) - m^2\phi(x) = 0 \rightarrow \ddot{\phi}(t,\vec{x}) - c^2\nabla^2\phi(t,\vec{x}) + m^2c^2\phi(t,\vec{x}) = 0$

* $\tilde{\phi}(t,\vec{k}) = \int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\phi(t,\vec{x}) \rightarrow \ddot{\tilde{\phi}}(t,\vec{k}) + \underbrace{[m^2c^2 + k^2c^2]}_{\equiv\,\omega^2}\tilde{\phi}(t,\vec{k}) = 0$

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R.P. Woodard

Solutions to Asmt #14

* $\tilde{\phi}(t,\vec{k}) = \tilde{\phi}_0(\vec{k})\cos(\omega t) + \frac{\dot{\tilde{\phi}}_0(\vec{k})}{\omega}\sin(\omega t)$

$$\rightarrow \phi^{(0)}(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \Big[\tilde{\phi}_0(\vec{k})\cos(\omega t) + \frac{\dot{\tilde{\phi}}_0(\vec{k})}{\omega}\sin(\omega t)\Big]$$

(3b) * exact eqn is $\ddot{\phi} - c^2\nabla^2\phi + c^2m^2\phi = -c^2\lambda\phi^3$

* eqn for 1st order is $\ddot{\phi}^{(1)} - c^2\nabla^2\phi^{(1)} + c^2m^2\phi^{(1)} = -c^2\lambda\big[\phi^{(0)}\big]^3$

* spatially Fourier-transforming gives $\ddot{\tilde{\phi}}^{(1)}(t,\vec{k}) + \omega^2\tilde{\phi}^{(1)}(t,\vec{k}) = -c^2\lambda\int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\big[\phi^{(0)}(t,\vec{x})\big]^3$

$$\rightarrow \tilde{\phi}^{(1)}(t,\vec{k}) = -c^2\lambda\int_0^t dt' \frac{\sin[\omega(t-t')]}{\omega}\int d^3x'\, e^{-i\vec{k}\cdot\vec{x}'}\big[\phi^{(0)}(t',\vec{x}')\big]^3$$

$$* \quad \phi^{(1)}(t,\vec{x}) = \int\frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \times -c^2\lambda\int_0^t dt' \frac{\sin[\omega(t-t')]}{\omega}\int d^3x'\, e^{-i\vec{k}\cdot\vec{x}'}\big[\phi^{(0)}(t',\vec{x}')\big]^3$$

* there is no point in performing the temporal integration