10/31/18
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Solutions to Asmt #11

(1a) * $\frac{dA}{dt} = \{A, H\} + \frac{\partial A}{\partial t} \Rightarrow \frac{\partial A}{\partial t} = -\{A, H\}$ & $\frac{dB}{dt} = 0 \Rightarrow \frac{\partial B}{\partial t} = -\{B, H\}$

* $\{\{A,B\},H\} = -\{\{B,H\},A\} - \{\{H,A\},B\} = \{\frac{\partial B}{\partial t}, A\} - \{\frac{\partial A}{\partial t}, B\} = -\frac{\partial}{\partial t}\{A,B\}$

$\Rightarrow \{\{A,B\},H\} + \frac{\partial}{\partial t}\{A,B\} = \frac{d}{dt}\{A,B\} = 0 \Rightarrow \{A,B\}$ is a constant of motion

(1b) * $A = H \Rightarrow \frac{\partial H}{\partial t} = 0$

* $\frac{dB}{dt} = \{H, B\} + \frac{\partial B}{\partial t} = 0 \Rightarrow \frac{\partial B}{\partial t} = -\{H, B\}$ is a constant of motion

* this can obviously be done as many times as desired $\Rightarrow \frac{\partial^n B}{\partial t^n}$ is a constant of motion

(1c) * Free particle Hamiltonian is $H = p^2/2m$

* $F(x,p,t) = x - \frac{pt}{m} \Rightarrow \frac{dF}{dt} = \{F, H\} + \frac{\partial F}{\partial t} = \{x - \frac{pt}{m}, \frac{p^2}{2m}\} - \frac{p}{m} = \frac{p}{m} - \frac{p}{m} = 0$

(2a) * I did this in class on Oct. 29 & the answer is

$$\Theta[A](t,x,y,z) = -\int_0^t dt'\, c A_0(t',x,y,z) + \int_0^x dx' A_1(0,x',y,z) + \int_0^y dy' A_2(0,0,y',z) + \int_0^z dz' A_3(0,0,0,z')$$

(2b) * $A_1'(t,x,y,z) = A_1(t,x,y,z) - \partial_x \Theta(t,x,y,z) = A_1(t,x,y,z) + \int_0^t dt'\, c\, \partial_x A_0(t',x,y,z) - A_1(0,x,y,z)$

$$= \int_0^t dt'\, \partial_{t'} A_1(t',x,y,z) - \int_0^t dt'\, c\, \partial_x A_0(t',x,y,z) = \int_0^t dt'\, c\, F_{01}(t',x,y,z)$$

(2c) * $A_2'(t,x,y,z) = \int_0^t dt'\, c\, F_{02}(t',x,y,z) + \int_0^x dx' F_{12}(0,x',y,z)$

* $A_3'(t,x,y,z) = \int_0^t dt'\, c\, F_{03}(t',x,y,z) + \int_0^x dx' F_{13}(0,x',y,z) + \int_0^y dy' F_{23}(0,0,y',z)$

(3) * Suppose A, B & C are each functions of $\vec{q}, \vec{p}$ & $t \Rightarrow$ just write out each PB & cancel

$$* \{\{A,B\},C\} = \sum_{i=1}^N \sum_{j=1}^N \Bigg[\frac{\partial^2 A}{\partial q_i \partial q_j}{}^{1} \frac{\partial B}{\partial p_i} \frac{\partial C}{\partial p_j} + \frac{\partial A}{\partial q_i} \frac{\partial^2 B}{\partial q_j \partial p_i}{}^{2} \frac{\partial C}{\partial p_j} - \frac{\partial^2 A}{\partial q_j \partial p_i}{}^{3} \frac{\partial B}{\partial q_i} \frac{\partial C}{\partial p_j} - \frac{\partial A}{\partial p_i} \frac{\partial^2 B}{\partial q_i \partial q_j}{}^{4} \frac{\partial C}{\partial p_j}$$
$$- \frac{\partial^2 A}{\partial q_i \partial p_j}{}^{5} \frac{\partial B}{\partial p_i} \frac{\partial C}{\partial q_j} - \frac{\partial A}{\partial q_i} \frac{\partial^2 B}{\partial p_i \partial p_j}{}^{6} \frac{\partial C}{\partial q_j} + \frac{\partial^2 A}{\partial p_i \partial p_j}{}^{7} \frac{\partial B}{\partial q_i} \frac{\partial C}{\partial q_j} + \frac{\partial A}{\partial p_i} \frac{\partial^2 B}{\partial q_i \partial p_j}{}^{8} \frac{\partial C}{\partial q_j} \Bigg]$$

$$* \{\{B,C\},A\} = \sum_{i=1}^N \sum_{j=1}^N \Bigg[\frac{\partial^2 B}{\partial q_i \partial q_j}{}^{4} \frac{\partial C}{\partial p_i} \frac{\partial A}{\partial p_j} + \frac{\partial B}{\partial q_i} \frac{\partial^2 C}{\partial q_j \partial p_i}{}^{9} \frac{\partial A}{\partial p_j} - \frac{\partial^2 B}{\partial q_j \partial p_i}{}^{8} \frac{\partial C}{\partial q_i} \frac{\partial A}{\partial p_j} - \frac{\partial B}{\partial p_i} \frac{\partial^2 C}{\partial q_i \partial q_j}{}^{10} \frac{\partial A}{\partial p_j}$$
$$- \frac{\partial^2 B}{\partial q_i \partial p_j}{}^{2} \frac{\partial C}{\partial p_i} \frac{\partial A}{\partial q_j} - \frac{\partial B}{\partial q_i} \frac{\partial^2 C}{\partial p_i \partial p_j}{}^{11} \frac{\partial A}{\partial q_j} + \frac{\partial^2 B}{\partial p_i \partial p_j}{}^{6} \frac{\partial C}{\partial q_i} \frac{\partial A}{\partial q_j} + \frac{\partial B}{\partial p_i} \frac{\partial^2 C}{\partial q_i \partial p_j}{}^{12} \frac{\partial A}{\partial q_j} \Bigg]$$

$$* \{\{C,A\},B\} = \sum_{i=1}^N \sum_{j=1}^N \Bigg[\frac{\partial^2 C}{\partial q_i \partial q_j}{}^{10} \frac{\partial A}{\partial p_i} \frac{\partial B}{\partial p_j} + \frac{\partial C}{\partial q_i} \frac{\partial^2 A}{\partial q_j \partial p_i}{}^{5} \frac{\partial B}{\partial p_j} - \frac{\partial^2 C}{\partial q_j \partial p_i} \frac{\partial A}{\partial q_i}{}^{12} \frac{\partial B}{\partial p_j} - \frac{\partial C}{\partial p_i} \frac{\partial^2 A}{\partial q_i \partial q_j}{}^{1} \frac{\partial B}{\partial p_j}$$
$$- \frac{\partial^2 C}{\partial q_i \partial p_j}{}^{9} \frac{\partial A}{\partial p_i} \frac{\partial B}{\partial q_j} - \frac{\partial C}{\partial q_i} \frac{\partial^2 A}{\partial p_i \partial p_j}{}^{7} \frac{\partial B}{\partial q_j} + \frac{\partial^2 C}{\partial p_i \partial p_j}{}^{11} \frac{\partial A}{\partial q_i} \frac{\partial B}{\partial q_j} + \frac{\partial C}{\partial p_i} \frac{\partial^2 A}{\partial q_i \partial p_j}{}^{3} \frac{\partial B}{\partial q_j} \Bigg]$$

* all 24 terms cancel $\Rightarrow \{\{A,B\},C\} + \{\{B,C\},A\} + \{\{C,A\},B\} = 0$