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Solutions to Asmt #1

① * rod's center $\vec{A}(\alpha) = a\cos(\alpha(t))\hat{x} + a\sin(\alpha(t))\hat{y}$

* rod's top-center $\vec{B}(\theta,\phi) = \frac{\ell}{2}\left[\sin(\theta(t))\cos(\phi(t))\hat{x} + \sin(\theta(t))\sin(\phi(t))\hat{y} + \cos(\theta(t))\hat{z}\right]$

2 particle positions

* $\vec{r}_1(t) = \vec{A}(\alpha(t)) + \vec{B}(\theta(t),\phi(t))$

* $\vec{r}_2(t) = \vec{A}(\alpha(t)) - \vec{B}(\theta(t),\phi(t))$

$$\left.\begin{aligned} &* \dot{\vec{r}}_1\cdot\dot{\vec{r}}_1 = \dot{\vec{A}}\cdot\dot{\vec{A}} + 2\dot{\vec{A}}\cdot\dot{\vec{B}} + \dot{\vec{B}}\cdot\dot{\vec{B}} \\ &* \dot{\vec{r}}_2\cdot\dot{\vec{r}}_2 = \dot{\vec{A}}\cdot\dot{\vec{A}} - 2\dot{\vec{A}}\cdot\dot{\vec{B}} + \dot{\vec{B}}\cdot\dot{\vec{B}} \end{aligned}\right\} \Rightarrow \|\dot{\vec{r}}_1\|^2 + \|\dot{\vec{r}}_2\| = 2\|\dot{\vec{A}}\|^2 + 2\|\dot{\vec{B}}\|^2$$

$$\therefore \boxed{KE = \tfrac{1}{2}m\left[\|\dot{\vec{r}}_1\| + \|\dot{\vec{r}}_2\|\right] = ma^2\dot{\alpha}^2 + \frac{m\ell^2}{4}\dot{\theta}^2 + \frac{m\ell^2}{4}\sin^2(\theta)\dot{\phi}^2}$$

② * $\frac{\partial L}{\partial \dot{x}} = \frac{1}{3}m^2\dot{x}^3 + 2m\dot{x}V \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m^2\dot{x}^2\ddot{x} + 2m\ddot{x}V + 2m\dot{x}^2V'$

* $\frac{\partial L}{\partial x} = m\dot{x}^2V' - 2VV'$

ⓐ $\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = m^2\dot{x}^2\ddot{x} + 2m\ddot{x}V + m\dot{x}^2V' + 2VV' = (m\dot{x}^2 + 2V)(m\ddot{x} + V') = 0$

ⓑ Assuming $V > 0 \rightarrow m\dot{x}^2 + 2V > 0 \rightarrow$ must have $m\ddot{x} + V' = 0$

* NB this is the eqn for a particle of mass m moving in potential $V(x)$

③ * $\frac{\partial L}{\partial \dot{q}} = e^{\gamma t}m\dot{q} \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = e^{\gamma t}(m\ddot{q} + \gamma m\dot{q})$

* $\frac{\partial L}{\partial q} = -e^{\gamma t}kq$

ⓐ $\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = e^{\gamma t}(m\ddot{q} + \gamma m\dot{q} + kq) = 0$

ⓑ this is a damped harmonic oscillator of mass m, spring constant k & damping factor γ

ⓒ $\left.\begin{aligned} &* \omega^2 \equiv k/m \\ &* q(t) \propto e^{i\Omega t} \end{aligned}\right\} \rightarrow -\Omega^2 + i\gamma\Omega + \omega^2 = 0 \rightarrow \Omega = \frac{i\gamma}{2} \pm \sqrt{\omega^2 - \frac{\gamma^2}{4}}$

$$q(t) = e^{-\frac{\gamma}{2}t}\left\{q_0\cos\left[\sqrt{\omega^2 - \tfrac{1}{4}\gamma^2}\,t\right] + \frac{[\dot{q}_0 + \frac{1}{2}\gamma q_0]}{\sqrt{\omega^2 - \frac{1}{4}\gamma^2}}\sin\left[\sqrt{\omega^2 - \tfrac{1}{4}\gamma^2}\,t\right]\right\}$$