PHY 6246 September 20, 2019

Assignment #6

Reading: Complete Chapter 5 and begin Chapter 6 in *Goldstein*.

Problems: Due by the start of class on Monday, 9/30/19.

(1) Consider a particle of air at latitude $90^\circ - \theta$ which is held at fixed altitude by the balance between the gravitational force and the pressure differential. Work in the rotating frame of the Earth, including the horizontal components of the Coriolis force but neglecting the centrifugal force.

(a) What is the angular velocity vector $\vec{\omega}$ assuming $\widehat{z}$ points upward and $\widehat{x}$ points East?

(b) Assuming the particle has velocity vector $\vec{v}(t) = v_x(t)\widehat{x} + v_y(t)\widehat{y}$, write down the x and y components of Newton's equation.

(c) Solve for $v_x(t)$ and $v_y(t)$, starting from arbitrary $v_x(0)$ and $v_y(0)$.

(d) Suppose the particle is originally moving East at latitude 30° North, how long does it take before the particle is moving South?

(2) Consider a cube of side length L and mass M which is rotating about one of its edges.

(a) What is the moment of inertia of the cube?

(b) Suppose the cube starts from rest, balanced on a flat surface on one of its edges, and falls onto the surface. What is the kinetic energy of the cube just before impact?

(c) What is the cube's angular velocity just before impact?

(3) Two carts of masses m_1 and m_2 are connected to right and left walls, and to eachother, by three parallel springs (with spring constants k_1, k_2 and k_3) so that the carts move in one dimension. From the left wall to the right wall the sequence of springs and carts is: k_1, m_1, k_2, m_2 and k_3.

(a) Write the dynamical equations in matrix form using the 2-component column vector comprised of the carts' deviations $x_1(t)$ and $x_2(t)$ from equilibrium.

(b) Let $m_1 = m$, $m_2 = 2m$, $k_1 = k$, $k_2 = 2k$ and $k_3 = 3k$. What are the characteristic frequencies of this system?

(c) What are the eigenvectors for each of the characteristic frequencies?