PHY 6246 September 5, 2019

**Assignment #3**

Reading: Complete Chapter 3 in *Goldstein*.

Problems: Due by the start of class on Monday, 9/9/19.

(1) Consider the Keplerian orbit,

$$r(\phi) = \frac{c}{1+\epsilon\cos(\phi)} \; .$$

(a) For the case $0 < \epsilon < 1$ show that the equation can be expressed in Cartesian coordinates as,

$$\left(\frac{x+d}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \; ,$$

and find the constants $a$, $b$ and $d$ in terms of $\epsilon$ and $c$.

(b) For the case of $\epsilon = 1$ show that the orbit describes a parabola in Cartesian coordinates and find the intercepts on the $x$ and $y$ axes.

(c) For the case $1 < \epsilon < \infty$ show that the equation can be expressed in Cartesian coordinates as,

$$\left(\frac{x+d}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1 \; ,$$

and find the constants $a$, $b$ and $d$ in terms of $\epsilon$ and $c$.

(2) A particle of mass $m$ moves in potential $V(r) = k/r$, where $k < 0$. This problem compares parabolic and circular orbits with the same angular momentum $L$.

(a) What is the ratio of the perihelion of the parabolic orbit to the radius of the circular orbit?

(b) What is the ratio of the speed of a particle in the parabolic orbit to the speed in the circular orbit when the particles have the same radius?

(3) Consider a particle of mass $m$ which moves in the Yukawa potential,

$$V(r) = -\frac{k}{r}\exp\Big[-\frac{r}{a}\Big] \; ,$$

where both $k$ and $a$ are positive constants.

(a) Reduce the equations of motion to the equivalent one-dimensional problem.

(b) Discuss the nature of orbits for different values of the energy and angular momentum.

(c) Suppose the orbit is nearly circular with radius $R$. Show that the apsides advance approximately $\pi R^2/a^2$ per revolution.