PHY 6246 August 26, 2019

Assignment #2

Reading: Complete Chapter 2 and begin Chapter 3 in *Goldstein*.

Problems: Due (in my office or mailbox) by noon on Tuesday, 9/3/19.

(1) Consider a point particle whose position is $q(t)$ and whose Lagrangian depends upon second as well as first time derivatives, $L(q, \dot{q}, \ddot{q}, t)$.

(a) Use Hamilton's Principle to find the Euler-Lagrange equation.

(b) Consider the energy function,

$$E = \dot{q}\Big[\frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}}\Big] + \ddot{q}\frac{\partial L}{\partial \ddot{q}} - L \; .$$

Use the Euler-Lagrange equation to show that E is constant if the Lagrangian has no explicit dependence upon time.

(c) The energy defined above can depend in a complicated way upon q, $\dot{q}$ and $\ddot{q}$, but it has a very simple dependence upon the 3rd time derivative of q. Exhibit this dependence.

(2) Consider the action functional,

$$S[q] = \int_0^T dt \Big[\tfrac{1}{2} m \dot{q}^2 - \tfrac{1}{2} k q^2\Big] \; .$$

(a) Recall the theorem that any function $q(t)$ which obeys $\dot{q}(0) = 0 = \dot{q}(T)$ can be expanded in a Fourer cosine sieries,

$$q(t) = \sum_{n=0}^{\infty} a_n \cos\Big(\frac{n\pi t}{T}\Big) \; .$$

Express $S[q]$ as a function of the Fourier coefficients a_n.

(b) Use Hamilton's Principle to derive equations for the coefficients a_n.

(c) Substitute the Fourier series expansion for $q(t)$ into the usual Euler-Lagrange equation and use the independence of the Fourier coefficents to infer equations for the a_n.

(3) Suppose a particle of mass m is subject to a central force such that its orbit is a perfect circle of radius R which passes through the origin ($r = 0$) of the central potential $V(r)$.

(a) Let $\alpha(t)$ represent the angle of the particle's orbit around the orbital center. What is the angular momentum in terms of m, R, α and $\dot{\alpha}$?

(b) What is the energy in terms of the same variables?

(c) Use the fact that both the angular momentum L and the energy E are constant to show that the potential must take the form $V(r) = \frac{k}{r^4}$, and find the constant k in terms of m, R, L and E.

(d) What is the particle's oribital period?