Assignment #14

Reading: Review Chapters 7-10 and 12-13 in *Goldstein*.

Problems: Due by the start of class on Monday, 12/02/19.

(1) Suppose the metric is described by two functions $A(r)$ and $B(r)$ of the radius r,

$$ds^2 = -B(r)c^2dt^2 + A(r)d\vec{x} \cdot d\vec{x} \; .$$

(a) Compute the nonzero components of the affine connection.
(b) Compute the nonzero components of the Riemann tensor.
(c) Compute the nonzero components of the Ricci tensor.
(d) Compute the Ricci scalar.
(e) Show that the source-free Einstein equations are satisfied by,

$$A(r) = \left(1 + \frac{K}{r}\right)^4 \qquad , \qquad B(r) = \frac{1}{A(r)}\left[1 - \left(\frac{K}{r}\right)^2\right]^2 \; .$$

(2) Recall the Yang-Mills Lagrange density,

$$\mathcal{L} = -\tfrac{1}{4}F_{a\mu\nu}F_a{}^{\mu\nu} \qquad , \qquad F_{a\mu\nu} \equiv \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - g f_{abc} A_{b\mu} A_{c\nu} \; .$$

(a) What is the Euler-Lagrange equation?
(b) Give the Hamiltonian in temporal gauge ($A_{a0} = 0$).
(c) What is the constraint equation?

(3) Consider a real scalar field $\phi(t, \vec{x})$ whose Lagrange density is,

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{4}\lambda\phi^4 \; .$$

(a) What is the general free field solution?
(b) What is the first order correction in λ?