Assignment #13

Reading: Chapter 13 in *Goldstein*.

Problems: Due by the start of class on Monday, 11/25/19.

(1) Consider a pendulum of mass m which is suspended on a massless string of length ℓ and released from rest at an angle θ_0 with respect to the vertical.

(a) Use action-angle variables to derive an exact expression for the frequency $f(\theta_0)$.

(b) Show that $f(0) = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$.

(c) Compute the first correction to $f(\theta_0)$ for small θ_0.

(2) Consider a higher derivative oscillator $q(t)$ whose Lagrangian is,

$$L = -\frac{gm}{2\omega^2}\ddot{q}^2 + \frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2 q^2 \ .$$

(a) Give the exact initial value solution in terms of q_0 and the inital values of its first three derivatives.

(b) Use perturbation theory in g to formulate a perturbative initial value solution in terms of just q_0 and $\dot{q}_0$. Obtain explicit results for $q^{(0)}(t)$ and $q^{(1)}(t)$.

(c) Which two of the four exact solutions in part (a) are present in the perturbative solution of part (b)?

(3) Consider an anharmonic oscillator whose lagrangian is,

$$L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2 q^2 - \frac{1}{4}\lambda q^4 \ ,$$

where λ is a positive constant with the dimension of energy over the 4th power of length.

(a) Use action-angle variables to derive an exact expression for the frequency f.

(b) Expand your result to first order in the small parameter $\frac{\lambda E}{m^2\omega^4}$.

(c) Use time-dependent perturbation theory to solve for $q(t)$ to first order in λ.