Assignment #12

Reading: Chapter 12 in *Goldstein*.

Problems: Due by the start of class on Monday, 11/18/19.

(1) One end of a uniform rod of length 2ℓ and mass m rests against a smooth horizontal floor, with the other end leaning against a smooth vertical surface. If the rod moves under the influence of gravity, with its ends in contact with the two frictionless surfaces, use the Hamilton-Jacobi equations to reduce the problem to quadratures.

(2) Consider a 3-dimensional particle of mass m which moves a conservative force. Set up the Hamilton-Jacobi equation in ellipsoidal coordinates u, v and ϕ defined by transforming from cylindrical coordinates,

$$\rho = a\sinh(v)\sin(u) \qquad , \qquad z = a\cosh(v)\cos(u) \ .$$

(a) For what forms of the potential $V(u, v, \phi)$ is the equation separable?

(b) Reduce to quadratures the case for which the potential is caused by the gravitational force of masses m_1 and $m_2 \neq m_1$, which are fixed on the z-axis at $z = \pm a$.

(3) This problem concerns 2-dimensional motion in the uniform gravitational field of the Earth where the y coordinate describes up and down, and the x coordinate describes left and right. Consider a particle of mass m which is constrained to move on the curve,

$$x(\lambda) = \ell\Big(2\lambda + \sin(2\lambda)\Big) \qquad , \qquad y(\lambda) = \ell\Big(1 - \cos(2\lambda)\Big) \ .$$

Use action-angle variables to find the frequency of oscillation for all initial conditions such that the parameter λ always obeys $\lambda \leq \frac{1}{4}\pi$.