Assignment #11

Reading: Chapter 10 in *Goldstein*.

Problems: Due by noon on Tuesday, 11/12/19.

(1) Consider a system with constants of motion $A(\vec{x}, \vec{p}, t)$ and $B(\vec{x}, \vec{p}, t)$.

(a) Prove that the Poisson bracket of A and B is a constant of motion, even when A and B depend explicitly on time.

(b) If A is the Hamiltonian, show that $\frac{\partial^n B}{\partial t^n}$ is also a constant of motion.

(c) Consider a free particle of mass m moving in one dimension. Show that $F = x - pt/m$ is a constant of motion.

(2) Many of us need to brush up our skills in multivariate calculus. A good exercise, which also conveys some important information, concerns enforcing a gauge condition in electrodynamics. Recall the 4-vector potential A_μ whose 4-curl gives the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. A *gauge transformation* of the vector potential is parameterized by an arbitrary function of space and time $\theta(t, \vec{x})$. The transformed field A'_μ is defined as,

$$A'_\mu(t, \vec{x}) \equiv A_\mu(t, \vec{x}) - \partial_\mu \theta(t, \vec{x}) \ .$$

Note that this transformation makes no change at all in the field strength tensor. You have probably seen in electrodynamics how gauge transformations can be used to impose conditions such as $\vec{\nabla} \cdot \vec{A} = 0$ (Coulomb gauge) or $\partial^\mu A_\mu = 0$ (Lorentz gauge) to simplify the field equations. In this problem we will impose a gauge condition known as *total temporal gauge.*

(a) Assuming $A_\mu(t, \vec{x})$ is an arbitrary function of space and time, solve for $\theta[A](t, \vec{x})$ as a functional of A_μ such that the transformed 4-vector potential obeys the conditions,

$$A'_0(t, x, y, z) = 0 \ \ , \ \ A'_1(0, x, y, z) = 0 \ \ , \ \ A'_2(0, 0, y, z) = 0 \ \ , \ \ A'_3(0, 0, 0, z) = 0 \ .$$

(b) Show that the x component of the vector potential is the following integral of the field strength tensor,

$$A'_1(t, x, y, z) = \int_0^t dt' c F_{01}(t', x, y, z) \ .$$

(c) It has been commented that "you can always make a gauge field invariant by defining it in a unique gauge". Illustrate the truth of this statement by expressing the y and z components of the transformed vector potential as integrals of the gauge invariant field strength tensor.

(3) Prove the Jacobi Identity for Poisson brackets.