PHY 6246 October 28, 2019

Assignment #10

Reading: Chapter 9 in *Goldstein*.

Problems: Due by the start of class on Monday, 11/04/19.

(1) Consider a system with Hamiltonian $H(q_1, q_2, p_1, p_2)$.

(a) Suppose we make the point transformation $Q_1 = q_1^2$ and $Q_2 = q_1 + q_2$. What are the most general transformations for the new momenta P_1 and P_2 such that the overall transformation is canonical?

(b) Apply your result to the Hamiltonian,

$$H = \frac{a}{2}\Big(\frac{p_1 - p_2}{2q_1}\Big)^2 + bp_2 + c(q_1 + q_2)^2 \ ,$$

where a, b and c are constants, to find a system in which Q_1 and Q_2 are ignorable.

(c) Use your result for (b) to obtain the full initial value solutions for $q_1(t)$, $q_2(t)$, $p_1(t)$ and $p_2(t)$.

(2) This problem concerns a nonrelativistic particle of mass m moving in a constant magnetic field $\vec{B} = B_0 \widehat{z}$.

(a) Show that the magnetic field derives from the vector potential $\vec{A} = \frac{1}{2} B_0(-y\widehat{x} + x\widehat{y})$.

(b) Write down the Lagrangian for this system and find the Hamiltonian.

(c) Prove that the following transformation is canonical where α is an arbitrary constant:

$$x = \frac{1}{\alpha}\Big(\sqrt{2P_1}\,\sin(Q_1) + P_2\Big) \quad , \quad p_x = \frac{\alpha}{2}\Big(\sqrt{2P_1}\,\cos(Q_1) - Q_2\Big) \ ,$$
$$y = \frac{1}{\alpha}\Big(\sqrt{2P_1}\,\cos(Q_1) + Q_2\Big) \quad , \quad p_y = \frac{\alpha}{2}\Big(-\sqrt{2P_1}\,\sin(Q_1) + P_2\Big) \ .$$

(d) Make the transformation with an appropriate choice of α to make the new Hamiltonian cyclic in Q_1 and Q_2.

(e) Write down the general initial value solution for $x(t)$ and $y(t)$ in terms of x_0, $\dot{x}_0$, y_0 and $\dot{y}_0$.

(3) Consider a system whose Hamiltonian is $H = \frac{1}{2q^2} + \frac{1}{2}p^2 q^4$.

(a) What are the equations of motion for $q(t)$?

(b) Find a canonical transformation to $K = \frac{1}{2}P^2 + \frac{1}{2}Q^2$.

(c) Show that the general initial value solution for (b) obeys (a).