PHY 6246 October 3, 2018

Exam #1

(1) Consider two particles of masses $m_1 = \frac{1}{3}M$ and $m_2 = \frac{2}{3}M$ which are attracted to one another by a potential $V(r) = \frac{1}{2}kr^2$ which depends on the square of the distance r between them.

(a) What is the reduced mass? **(16 points)**

(b) Reduce this system to a 1-dimensional problem for fixed angular momentum L and energy E. **(16 points)**

(c) What are the turning points? **(16 points)**

(d) Find the orbit $r(\phi)$. **(16 points)** It might help to change variables from r to $v = r^{-2}$, and to recall the integral

$$\int \frac{dv}{\sqrt{-v^2+2av-b}} = \sin^{-1}\left(\frac{v-a}{\sqrt{a^2-b}}\right) .$$

(e) Write down an integral (but do not evaluate it) for the orbital period. **(16 points)**

(2) Consider an ellipsoid of constant density ρ_0 which is bounded by the surface (in cylidrical coordinates),

$$\left(\frac{\rho}{a}\right)^2 + \left(\frac{z}{b}\right)^2 = 1 ,$$

where $a > b$ are positive constants.

(a) What is the total mass M and the center of mass $\vec{R}$? **(17 points)**

(b) What is the momentum of inertia tensor I_{ij}? **(17 points)**

(c) What are the principle axes and the associated moments? **(17 points)**

(d) Suppose the angular velocity along the $\widehat{z}$ axis is a constant ω (but the velocities in the x and y directions are not necessarily constant) and that there is no torque. What is the rate of precession? **(17 points)**

(e) What is the angular momentum vector in the body frame? **(17 points)**

(3) A massless spring, of force constant k_1 and unstrechted length ℓ_1, is suspended from the ceiling with a mass m_1 hanging from its lower end. A second spring, of force constant k_2 and unstretched length ℓ_2, is suspended from m_1, with a mass m_2 attached to its lower end. In this problem you are to include both the gravitational force and the spring force.

(a) Let $d_1(t)$ and $d_2(t)$ be the distances of the two masses from the ceiling. What is the Lagrangian? **(16 points)**

(b) What are the equilibrium values of d_1 and d_2? **(16 points)**

(c) What are the characteristic frequencies of this system? **(16 points)**

(d) What are the eigenvectors associated with each frequency? **(16 points)**

(e) Supposing that each mass is released from rest, what initial values $d_1(0)$ and $d_2(0)$ would lead to a solution oscillating at the samller of the two frequencies in part (c)? **(16 points)**