PHY 6246 October 2, 2019

Exam #1

(1) Consider particles of masses m_1 and m_2 that interact through the central potential which includes the lowest order general relativistic correction,

$$V(r) = -\frac{Gm_1m_2}{r}\left\{1 + \frac{3G(m_1+m_2)}{rc^2}\right\},$$

where c is the speed of light. **Do not include special relativistic effects**, just treat this as a central force problem.

(a) What are the total mass and the reduced mass? **(16 points)**

(b) Reduce this system to a 1-dimensional problem for fixed angular momentum L and energy E. **(16 points)**

(c) What are the turning points? **(16 points)**

(d) Find the orbit $r(\phi)$. **(16 points)** It might help to change variables from r to $u = r^{-1}$, and to recall the integral

$$\int \frac{du}{\sqrt{-u^2+2au-b}} = \sin^{-1}\left(\frac{u-a}{\sqrt{a^2-b}}\right).$$

(e) Find the advance of the apsides to lowest order about a circular orbit. **(16 points)**

(2) Consider a rigid body composed of two uniform rods, each of mass m and length L. The first rod lies on the x axis from $x = 0$ to $x = L$; the second rod lies on the y axis from $y = 0$ to $y = L$. Note that there are still **three** spatial dimensions, even though these rods both lie at $z = 0$.

(a) What is the total mass M and the center of mass $\vec{R}$? **(17 points)**

(b) What is the momentum of inertia tensor I_{ij} about its center? **(17 points)**

(c) What are the principle axes and the associated moments? **(17 points)**

(d) Suppose the angular velocity along the $\widehat{z}$ axis is held constant at ω by an applied torque, but that the velocities in the x and y directions (of the principal axes) are not constant, and that there is no torque in the $\widehat{x}$ or $\widehat{y}$ direction. What is the rate of precession? **(17 points)**

(e) What is the angular momentum vector in the body frame? **(17 points)**

(3) Consider a linear, triatomic molecule which is formed of masses (from left to right) $m_1 = m$, $m_2 = 2m$ and $m_3 = 3m$. The spring joining m_1 and m_2 has spring constant k and unstretched length a. The spring joining m_2 and m_3 has spring constant $3k$ and unstreched length b.

(a) What is the Lagrangian? **(16 points)**

(b) What are the equilibrium positions? **(16 points)**

(c) What are the characteristic frequencies? **(16 points)**

(d) What are the eigenvectors associated with each frequency? **(16 points)**

(e) Suppose that each mass is released from rest with $x_1(0) = L$, $x_2(0) = 2L$ and $x_3(0) = 3L$. What is the solution for $x_1(t)$? **(16 points)**